

Fiber and component metrology for high-speed communications:

What the manual doesn't tell you

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Boulder, Colorado

1. Polarization-mode dispersion (Williams)
2. Transmitter/receiver frequency response (Hale and Clement)

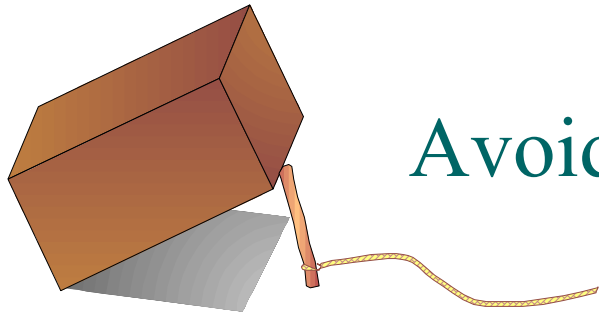
PMD measurement advice for folks with turnkey measurement systems

Assumptions:

A basic understanding of PMD

A PMD measurement system

An understanding of the measurement techniques

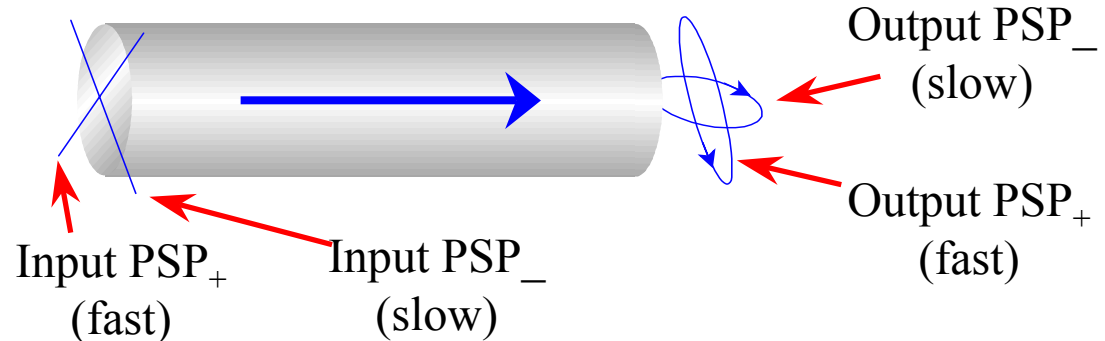


Avoid measurement traps that give false results.

1. Perform measurement “calibration”
2. Understand limitations imposed by measurement conditions
3. Choose measurement parameters correctly
4. Be aware of measurement uncertainties

Assumption: measurement system works correctly

General Case:

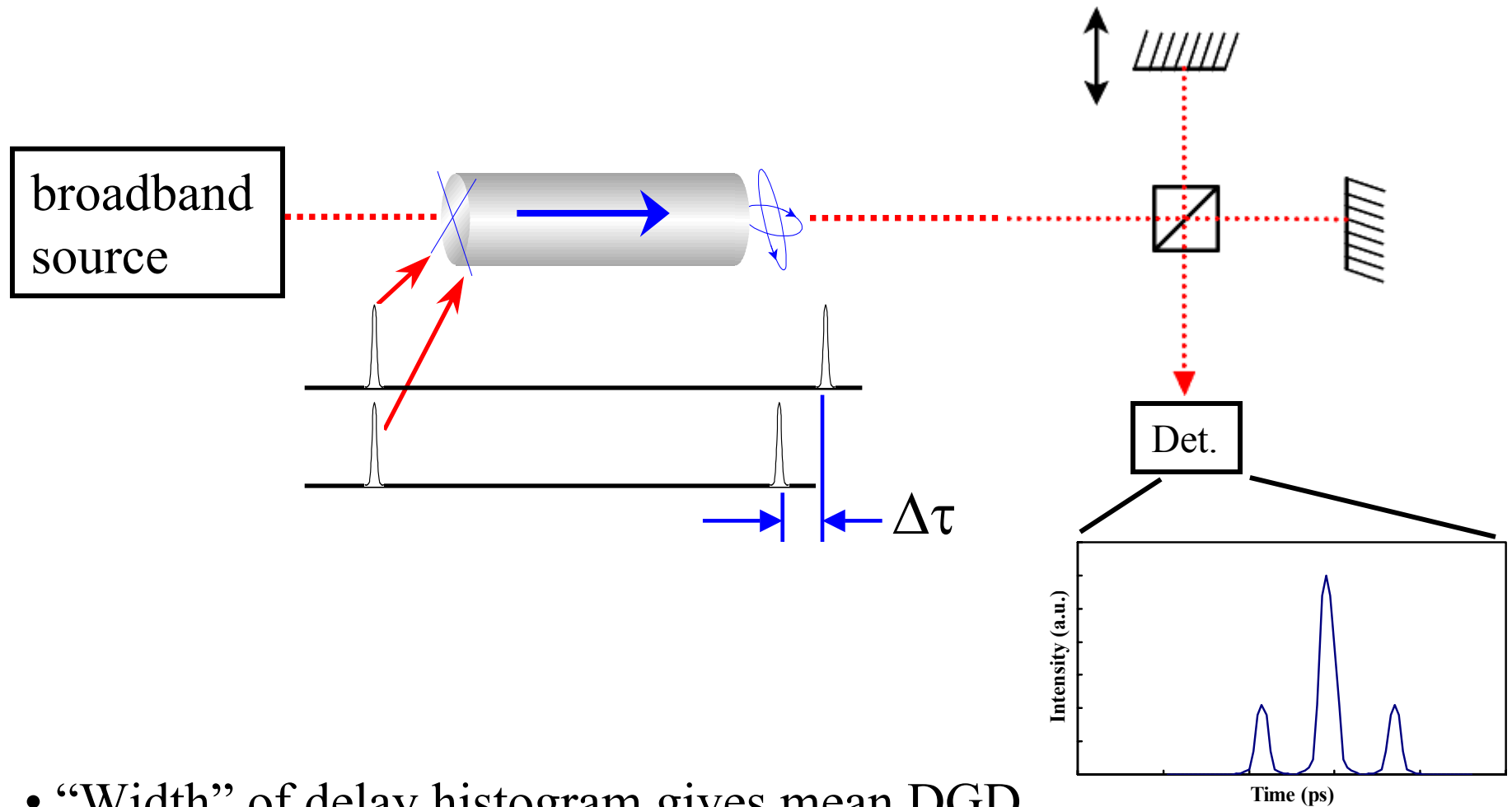


- Birefringence affects propagation velocity and output polarization state
- PMD is characterized by two Principal States of Polarization (PSP)
- PSPs are wavelength-independent (to first order)
- Propagation along PSPs is the fastest/slowest possible
- PMD is the phenomenon, DGD ($\Delta\tau$) is the magnitude

$$\text{Differential group delay (DGD): } \Delta\tau = \tau_{fast} - \tau_{slow}$$

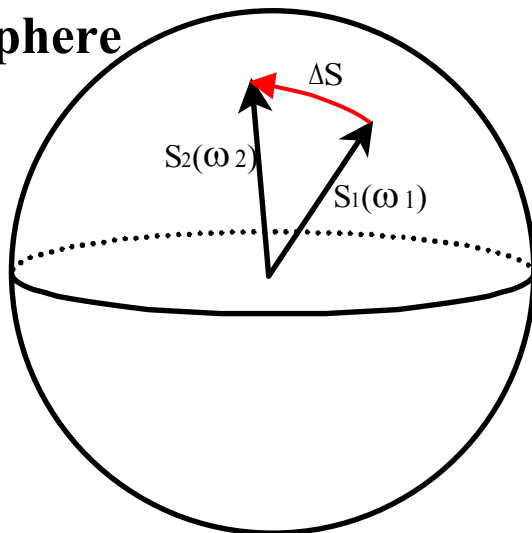
Review: Time domain PMD measurement

low-coherence interferometry (INT)



- “Width” of delay histogram gives mean DGD
- Measures only mean DGD

Poincaré sphere



(3-d representation of polarization state)

Equator: *linear polarization states*

Poles: *left and right circular*

Elsewhere: *elliptical*

Measure transmitted polarization state of light - at two optical frequencies (ω_1 and ω_2)

$$\Delta \tau = \left| \frac{d \vec{S}}{d \omega} \right| \quad \text{Differential Group Delay (DGD)}$$

Polarization-based DGD definition

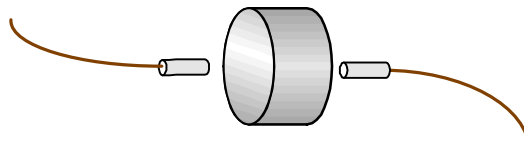
(ω = radian frequency)

Measures DGD(λ)

Jones Matrix Eigenanalysis (JME), Mueller matrix method (MMM), Poincaré sphere analysis (PSA)...

Review: Polarization-mode coupling

Non-mode-coupled devices:



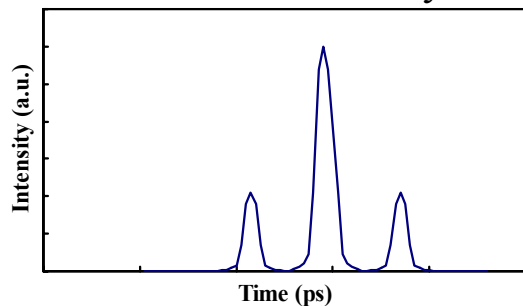
Simple birefringence (fast and slow eigenaxes independent of wavelength)

Examples:

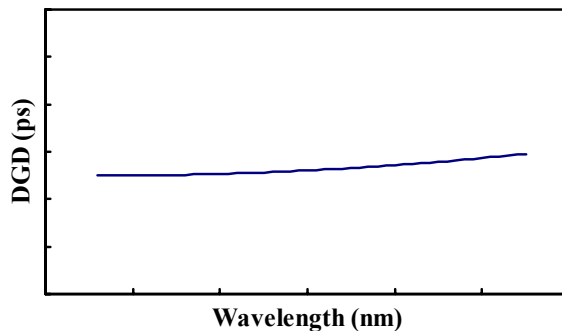
waveplates, single crystals, polarization maintaining fiber, typical components

Typical measurement results (non-mode-coupled)

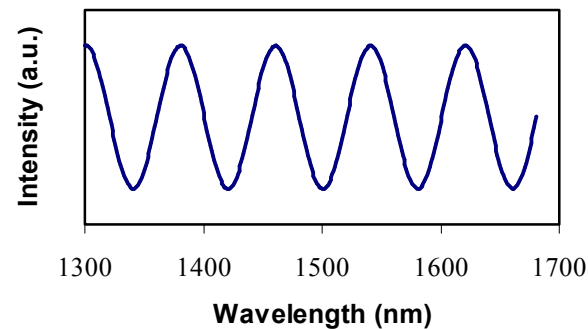
Low Coherence Interferometry



Polarimetric

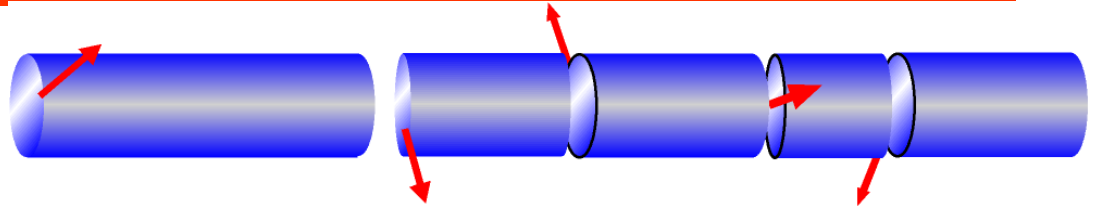


Fixed analyzer



Review: Polarization-mode coupling

Mode-coupled devices:



Complex birefringence (collection of simple birefringent elements)

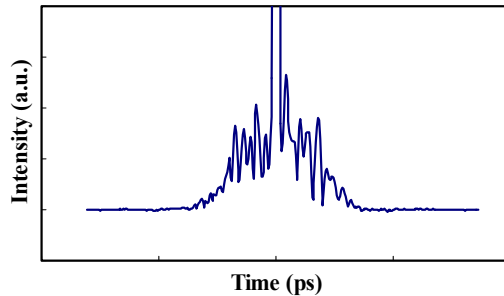
Fast and slow eigenaxes are wavelength-dependent

Examples:

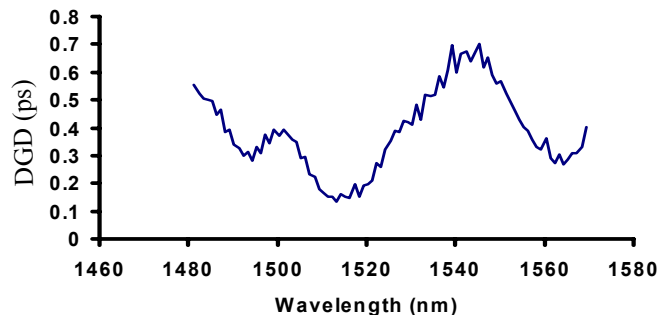
Long fibers, multiple splices of polarization maintaining fiber, full systems

Typical measurement results (Mode-coupled)

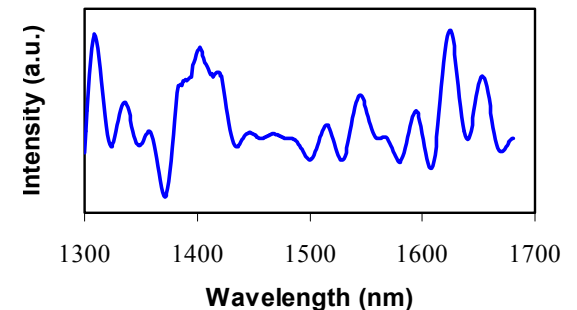
Low Coherence
Interferometry



Polarimetric



Fixed analyzer

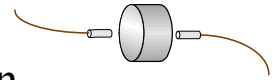


Artifact Selection Criteria:

- DGD approximates that of your DUT
- DGD can be predicted by other means
- Environmentally stable

Single birefringent crystal

- Predictable DGD $\Delta\tau = \frac{\Delta n_g L}{c}$, L =thickness, Δn_g = group birefringence
- Beware of waveplate tilt, multiple reflections, and dispersion
- Maximum DGD limited (0.5 ps or so)



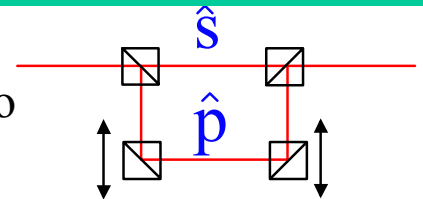
Polarization- maintaining fiber

- DGD predictable but less certain
- Beware of temperature coeff. (2-10x > quartz)
- Large DGD values possible



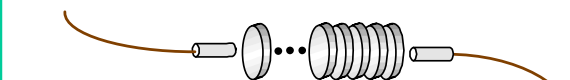
PMD emulator

- DGD predictable from geometry
- Beware of reflections and polarization extinction ratio
- Variable DGD possible

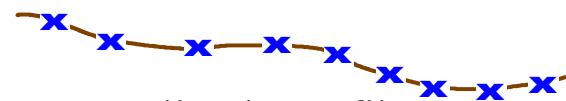


Artifact Selection Criteria:

- Mean DGD approximates that of your DUT
- $\text{DGD}(\lambda)$ “looks typical”
- Environmentally stable

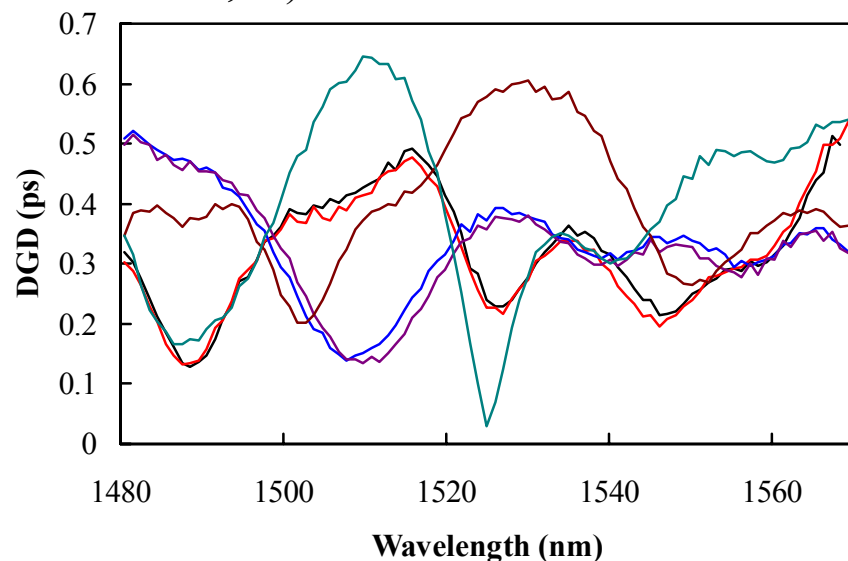


Stacked, misaligned crystals



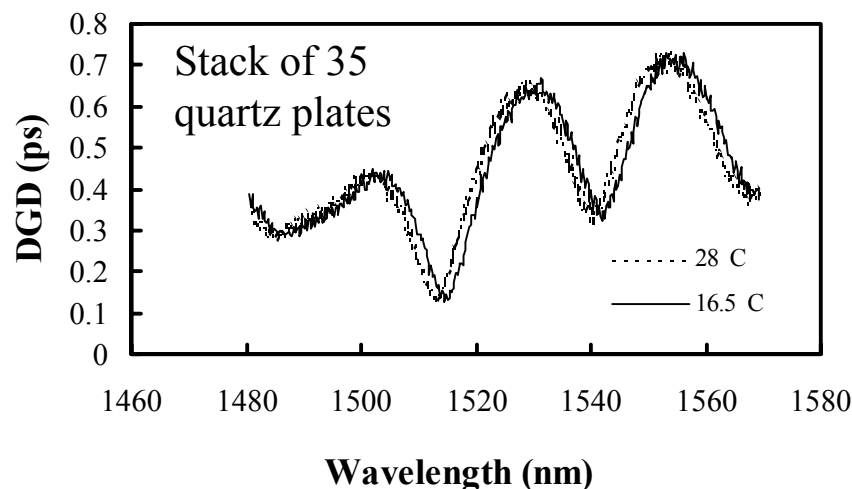
Spliced PM fiber

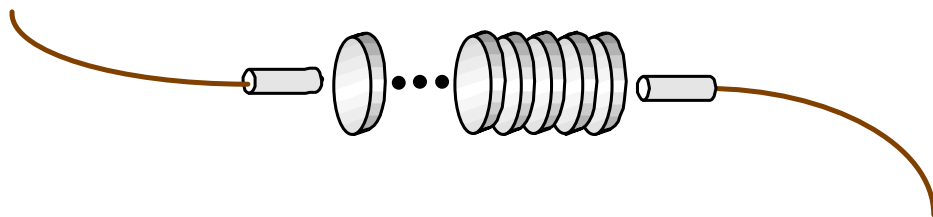
DGD prediction difficult (lenses, alignment, adhesives,...)



Multiple 35-plate stacks (identical preparation)

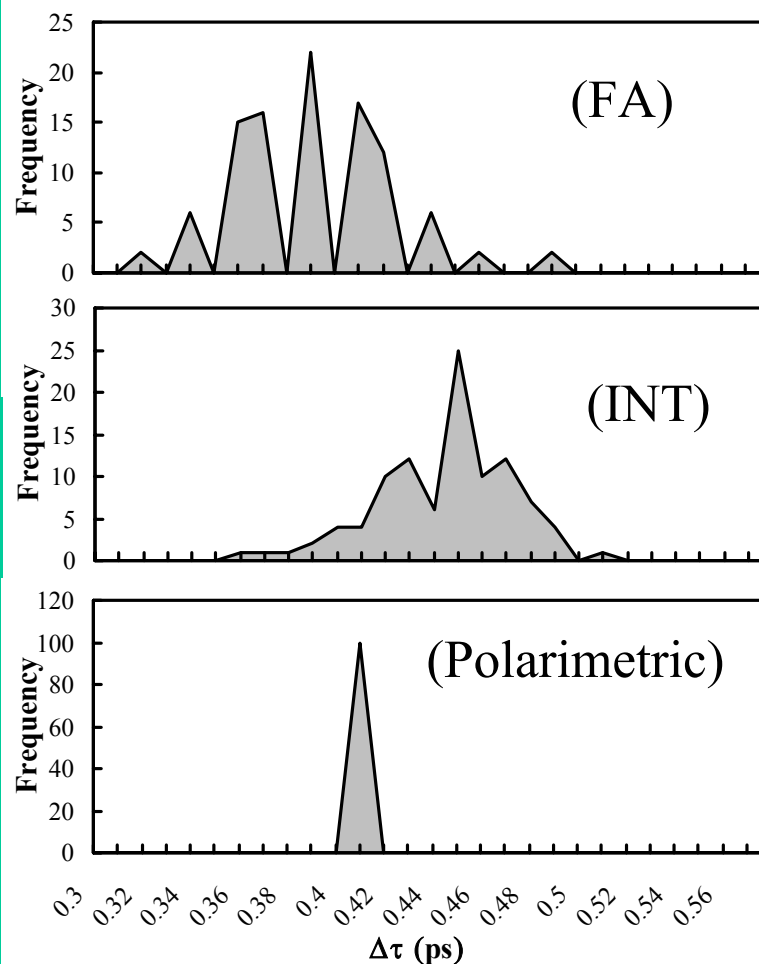
“wavelength shift” with temperature





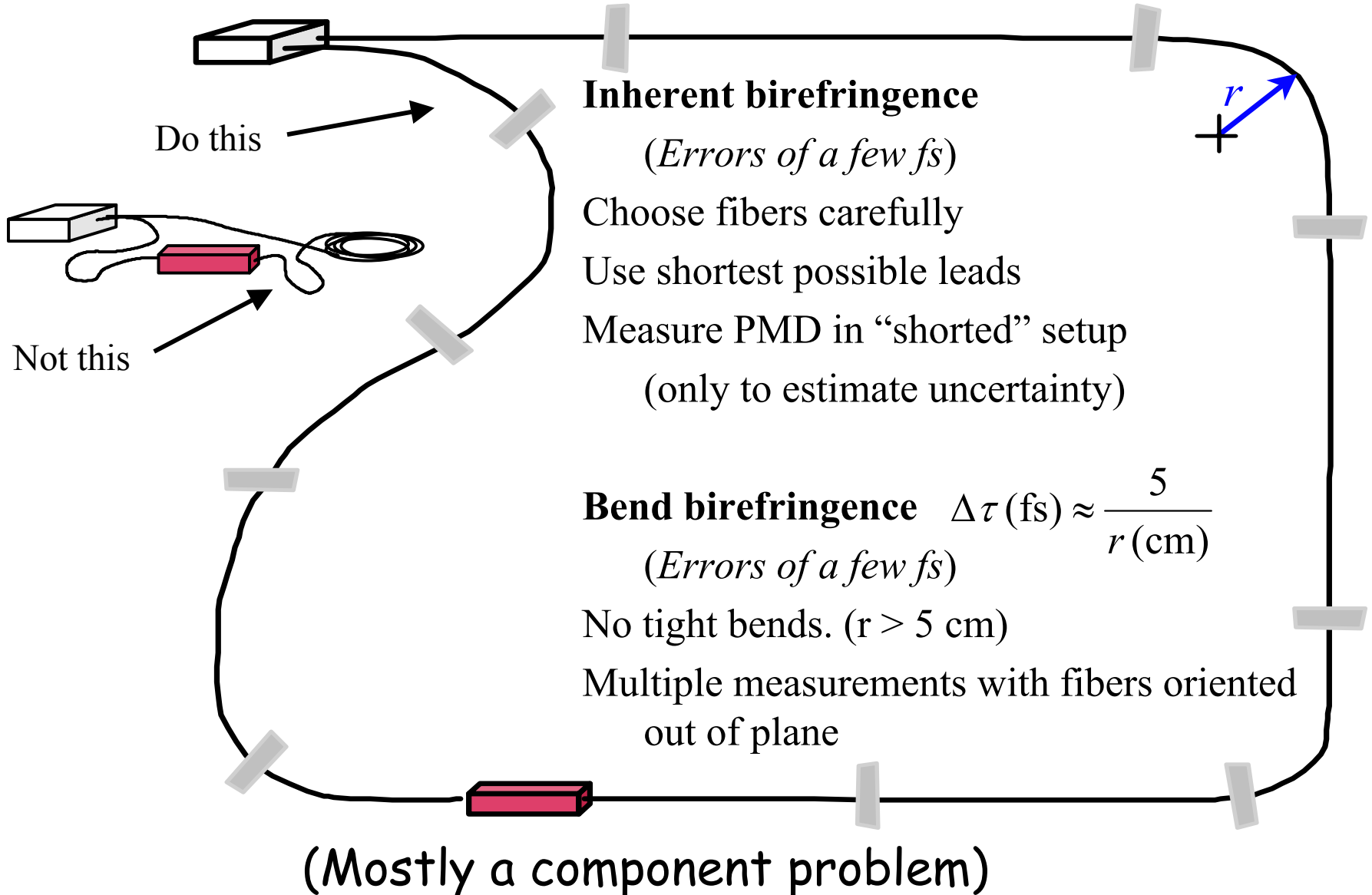
Beware:

- Instability with fiber lead reorientation (Non-polarimetric techniques)
- Disagreement between techniques



Simulation: 35 quartz plates, 200 nm measurement spectrum, 100 measurements

Limitations: fiber lead birefringence



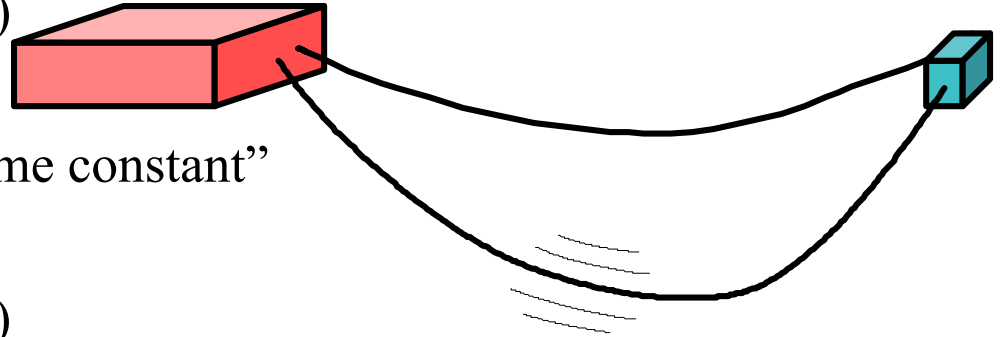
Limitations: dynamics of measurement path

Fiber leads moving (modulates polarization, adds noise)

(Errors proportional to total DGD)

Stabilize fibers

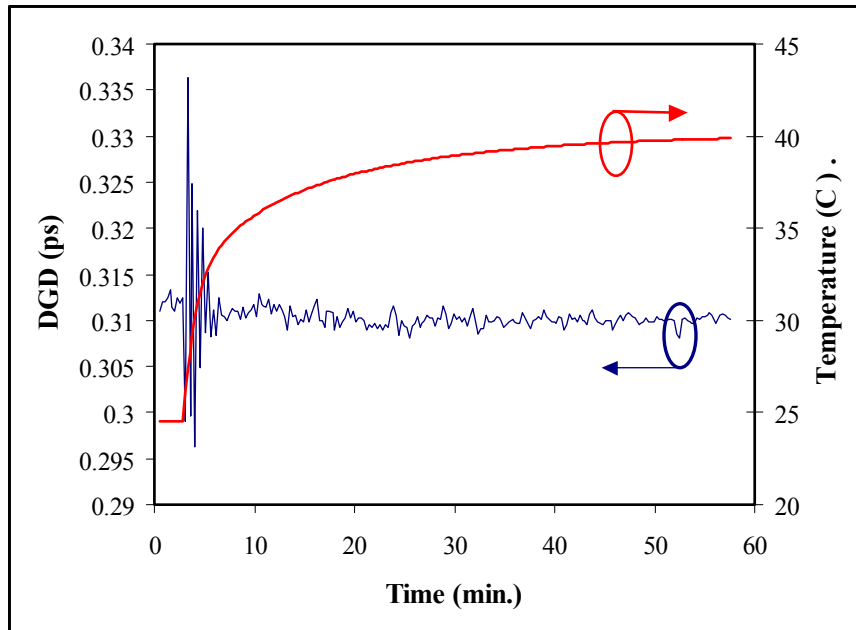
Keep motion slow compared to “time constant”



DUT thermal stability

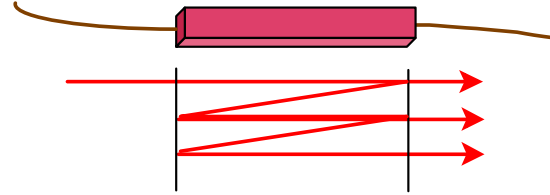
(Errors proportional to total DGD)

Interferometric technique less sensitive



Limitations: Multiple reflections

“Multi-path interference” (MPI)



Impact of MPI

DGD errors regardless of cavity geometry

Interferometry, Fixed analyzer (un-normalized)

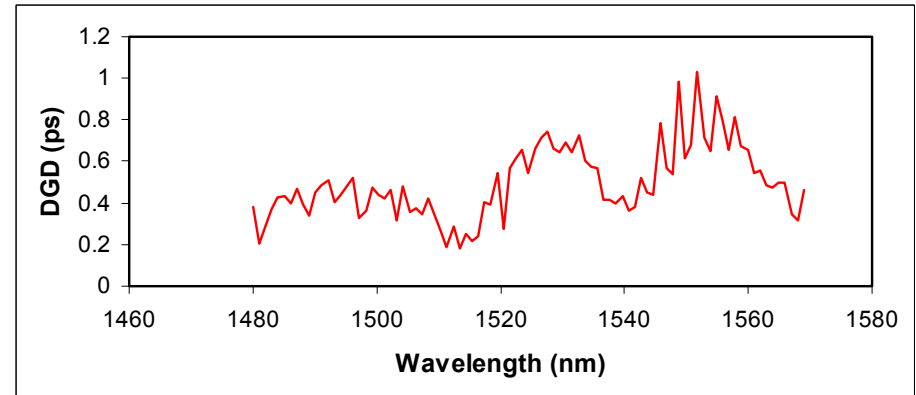
DGD errors only if PMD within cavity

MMM, JME, PSA, MPS, PSD...

Fixed analyzer (normalized)

DGD errors have zero mean

MMM, JME, PSA, MPS, PSD...



Mitigation of MPI (polarimetric techniques)

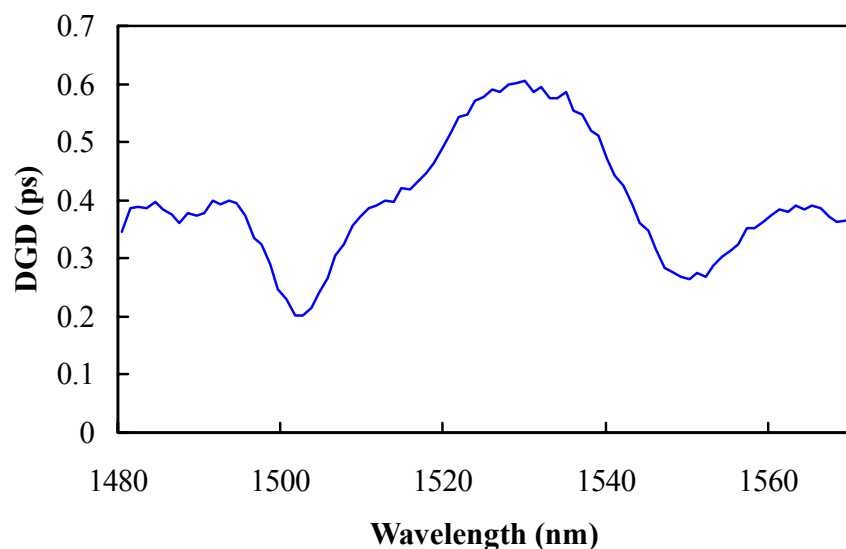
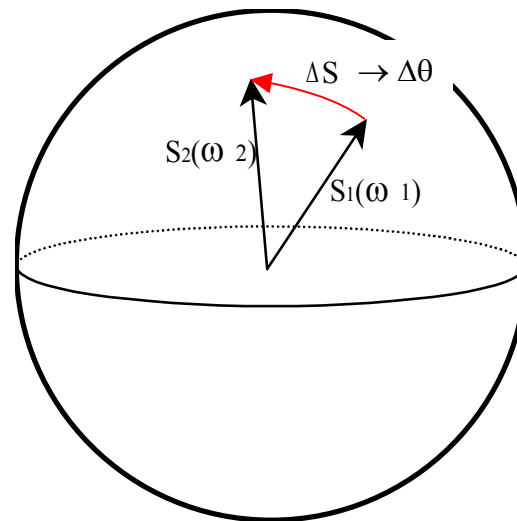
Average DGD over many ripple periods is stable

Make several measurements within each ripple period

Generic polarimetric measurement:

1. Launch light at two different wavelengths.
2. Measure the respective outputs S_1 & S_2 .
3. Calculate arc between S_1 and S_2 .
4. Calculate DGD.

$$\Delta\tau = \left| \frac{\Delta\theta}{\Delta\omega} \right|$$



Basic questions (JME, MMM, PSA...):

“Start/stop wavelength ?”

“Step size $\Delta\lambda$?”

$\Delta\lambda$ (or $\Delta\omega$) selects resolution

ΔS selects noise, DGD range, bias)

Parameters: Noise due to step size

DGD noise due to Stokes noise and wavelength jitter

$$\Delta\tau = \left| \frac{\Delta\theta}{\Delta\omega} \right|$$

$$\left(\frac{d(\Delta\tau)}{\Delta\tau} \right)^2 = \left(\frac{d(\Delta S)}{\Delta S} \right)^2 + \left(\frac{d(\Delta\omega)}{\Delta\omega} \right)^2$$

Relative DGD uncertainty

- With wavelength meter, $d(\Delta\omega)/\Delta\omega$ negligible
- Stokes noise $d(\Delta S)$ is fixed (for a given measurement setup)

(choose largest ΔS possible – large wavelength step)

$$\left(\frac{d(\Delta\tau)}{\Delta\tau} \right)^2 \approx \left(\frac{d(\Delta S)}{\Delta S} \right)^2$$

Parameters: Minimum step size (components)

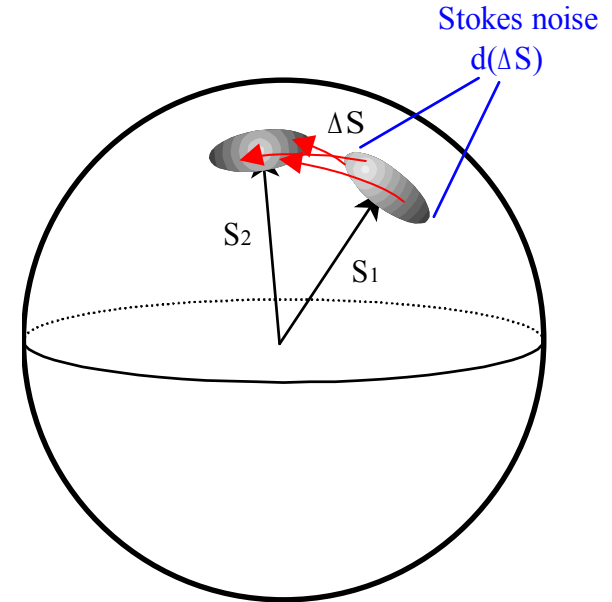
DGD noise due to Stokes noise

$$\left(\frac{d(\Delta\tau)}{\Delta\tau} \right)^2 \approx \left(\frac{d(\Delta S)}{\Delta S} \right)^2$$

$$\alpha \equiv \frac{1}{d(\Delta S)} = \text{“Bandwidth efficiency factor”}$$

$$\Delta\tau = \left| \frac{\Delta\theta}{\Delta\omega} \right|$$

$$\text{SNR} \leq \alpha \Delta\tau \Delta\omega$$



Example: JME measurement, $\alpha = 250$, filter BW = 50 GHz ($\Delta\omega = 3 \times 10^{11}$ rad/s)

Q. “What will be your SNR if you measure a 0.1 ps device?”

A. SNR = 8, or 13 % noise

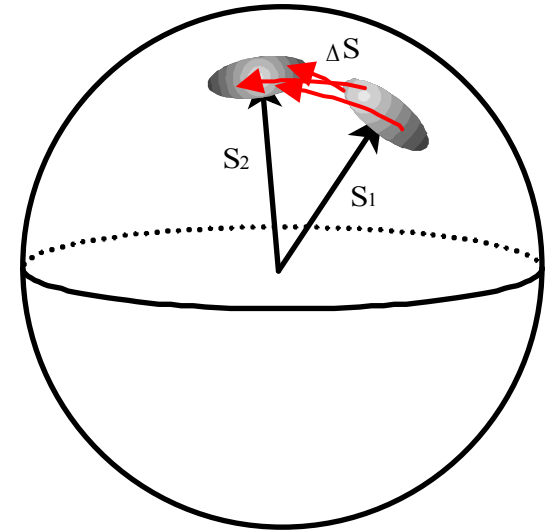
Parameters: Bandwidth efficiency factor α

	α (expression)	α “Typical”	Limiting factor
Low coherence interferometry (INT)	$0.75/\pi$	0.24	Gaussian source spectrum
Fixed Analyzer (FA)	$1/\pi$	0.3	Extrema spacing
Polarimetric (JME, PSA, MMM,...)	$1/d(\Delta S)$	250	Stokes noise, wavelength resolution
RF Phase-shift (MPS, PSD)	$\frac{360^\circ}{4\pi \Delta\varphi_{\text{deg}}}$	1400	Phase resolution (noise)

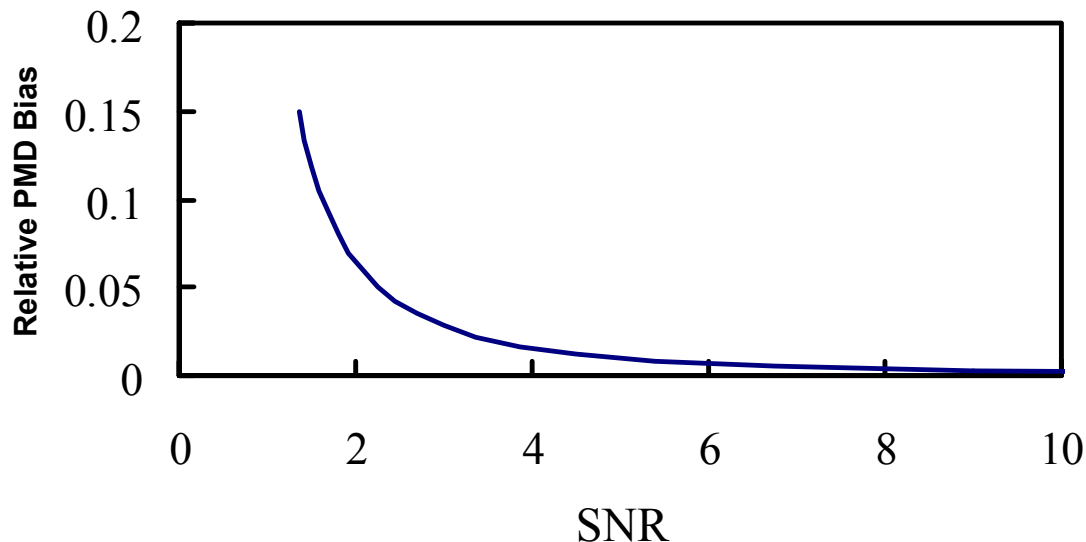
Parameters: Bandwidth efficiency bias

Measuring a small PMD in a narrow bandwidth can give a bias (not just random noise).

$$\text{SNR} \leq \alpha \Delta \tau \Delta \omega \quad \Delta \tau = \left| \frac{\Delta \theta}{\Delta \omega} \right|$$



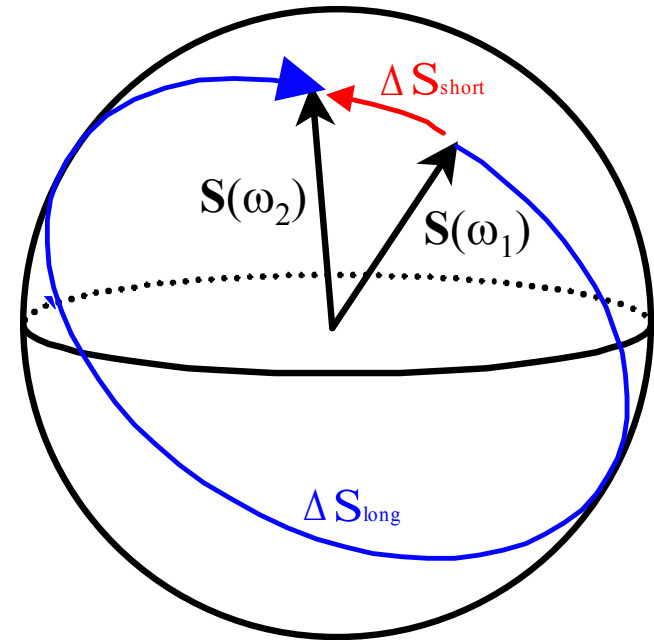
Bias for Polarimetric PMD Measurements



P.A. Williams, *Applied Optics*, **38**, 6508-6515 (1999).

$$\Delta\tau = \left| \frac{\Delta\theta}{\Delta\omega} \right| \quad \text{SNR} \leq \alpha \Delta\tau \Delta\omega$$

Better noise performance for larger $\Delta\omega$



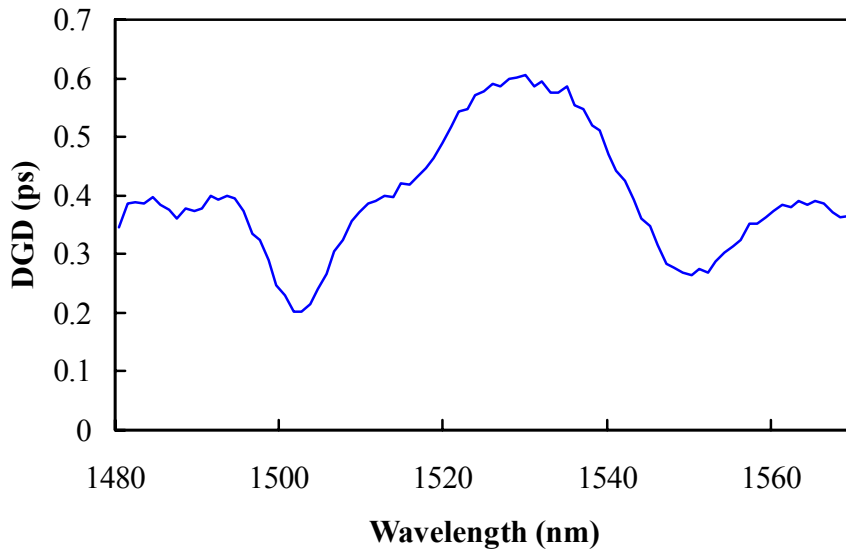
- ΔS is measured as the shortest path between $S(\omega_1)$ and $S(\omega_2)$
- Aliasing always under-reports DGD
- Rule of thumb: $\Delta\tau\Delta\omega < \pi$ or $\Delta\tau\Delta\lambda < 4 \text{ ps}\cdot\text{nm}$ (for 1550 nm)
- Reduce $\Delta\lambda$ and repeat measurement (is result the same?)

Parameters: 2nd-order PMD limits max. step size (even if you're measuring 1st order)

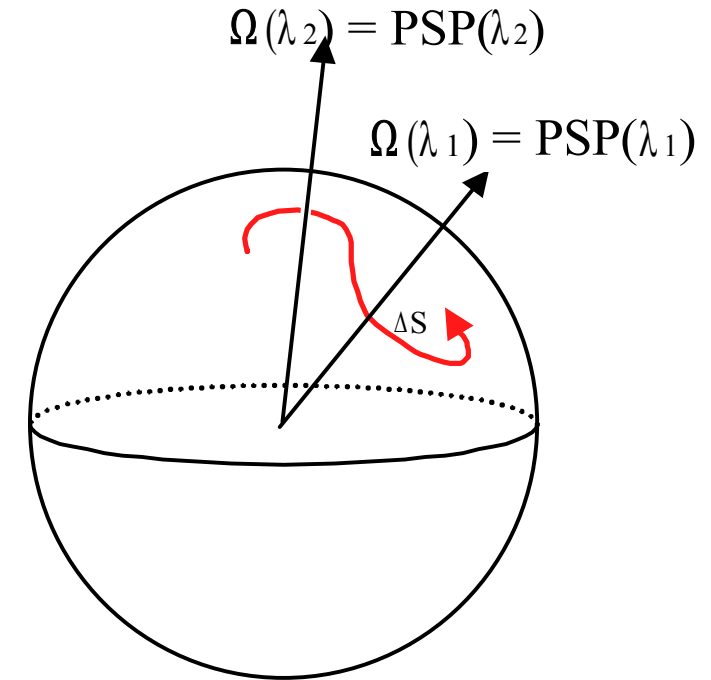
1st Order: $\vec{\Omega} = \Delta \tau \hat{s}$

2nd-Order: $\vec{\Omega}_\omega = \Delta \tau_\omega \hat{s} + \Delta \tau \hat{s}_\omega$

Polarization-dependent chromatic dispersion (PCD)



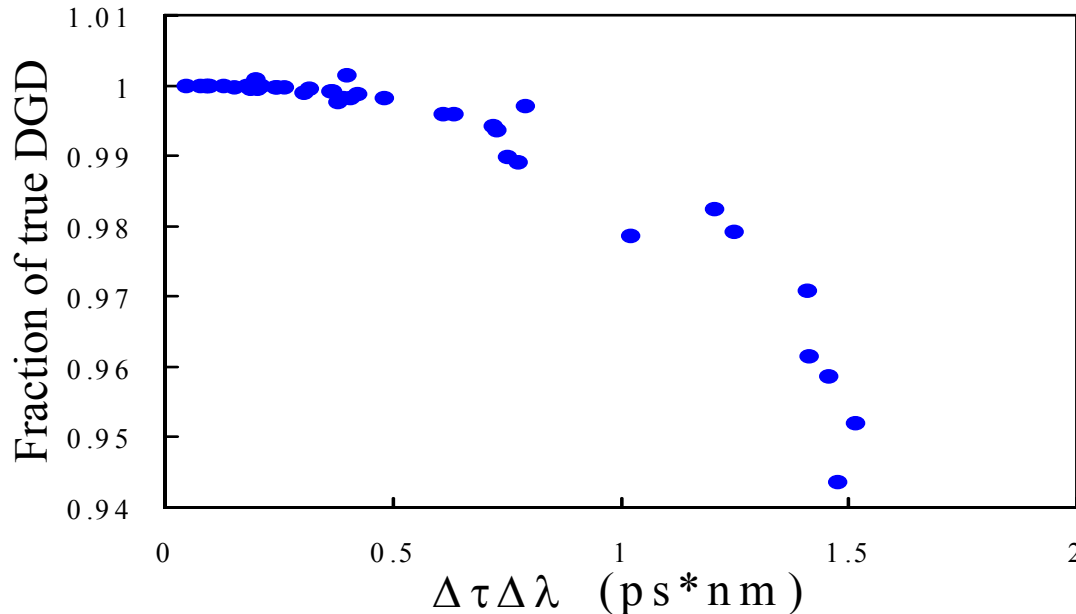
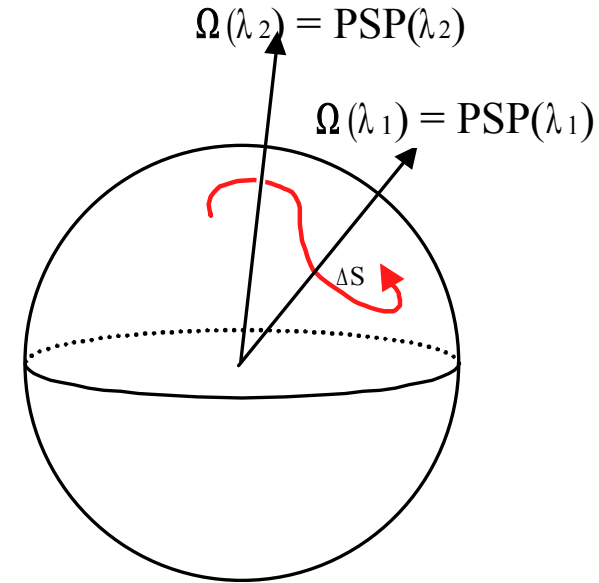
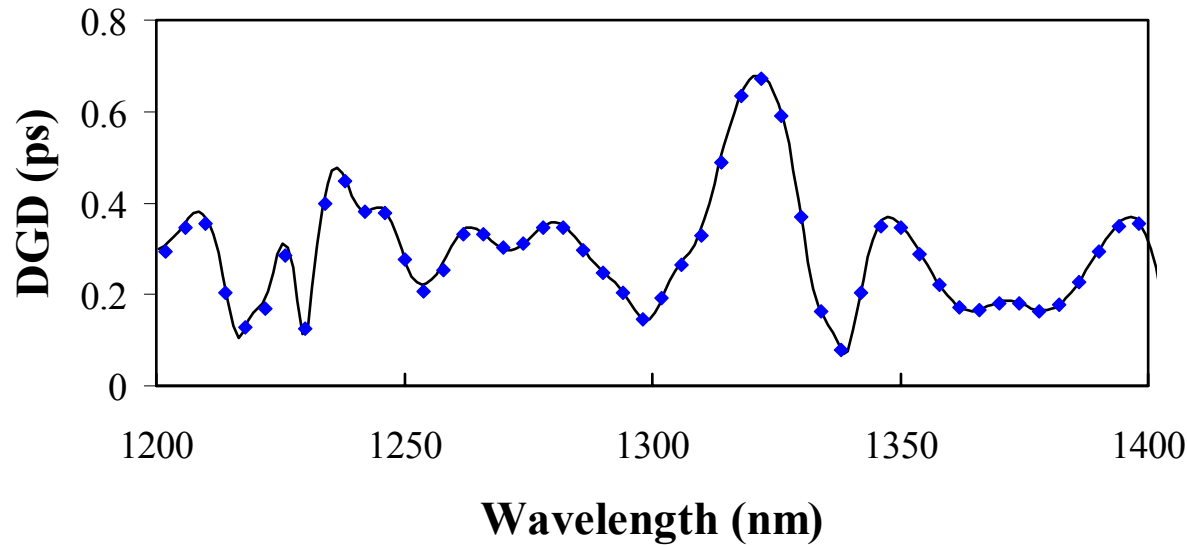
“Depolarization”
(wavelength-dependent PSP)



- Sample sufficiently to resolve DGD features
- Decrease wavelength step size to resolve a smooth curve (may be noisy)

Parameters: 2nd-order PMD limits max step size

(A smooth curve is not enough)



Simulation:

Highly mode-coupled devices.

Independent of mean DGD

For 5% accuracy:

$$\Delta\tau\Delta\lambda < 1.5 \text{ ps}\cdot\text{nm}$$

What's the dominant uncertainty source?

...Depends on the measurement.

Broad bandwidth or small PMD

Dominated by fiber lead birefringence uncertainty

$$\text{SNR} \leq \alpha \Delta \tau \Delta \omega$$

Example:

MMM technique, $\alpha=250$, step size = 10 nm,
DGD = 0.1 ps

SNR=196 (noise = 0.5% or 0.5 fs)

Narrow bandwidth or big PMD

Dominated by random noise (Bandwidth efficiency factor, α)

Example:

MMM technique, $\alpha=250$, step size = 0.1 nm, DGD = 2 ps

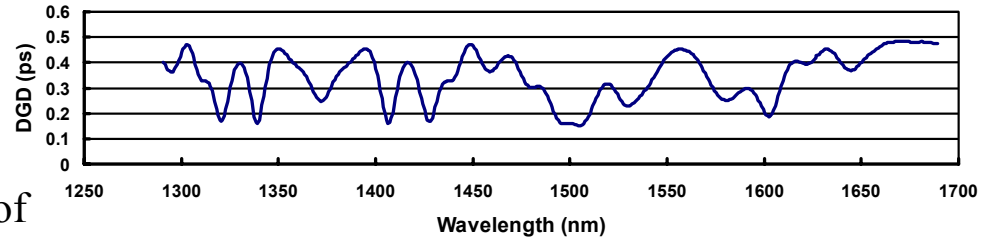
SNR=39 (noise = 2.6 % or 51 fs)

Uncertainty: Mode-coupled fibers

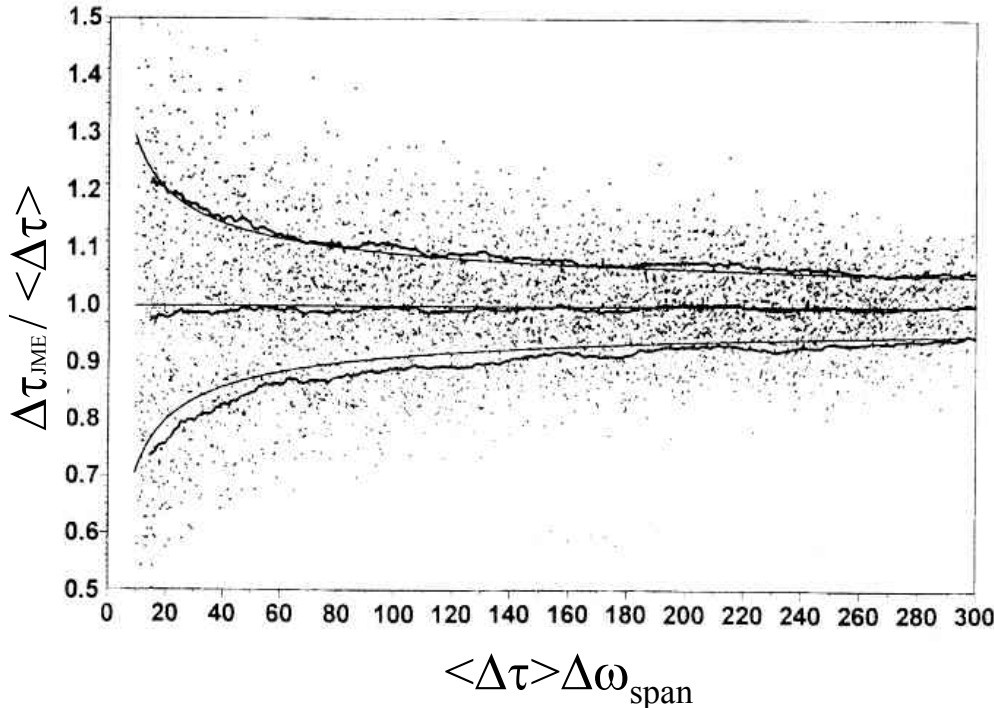
Mode-coupled fibers

Dominated by statistical uncertainty

$$\frac{\sigma}{\langle \Delta \tau \rangle} \approx \frac{1}{\sqrt{\langle \Delta \tau \rangle \Delta \omega_{span}}} \leftarrow \text{Total span of measurement}$$



JME : Measured PMD / Theoretical Mean PMD



Example: DGD=5 ps, span = 25 nm

$\sigma / \langle \Delta \tau \rangle$	# measurements required
10 %	1
5 %	4
1 %	100

Multiple measurements must be statistically independent - (time, temperature, re-arranged fibers).

- **General PMD Review**

1. Edward Collette, Ed., *Polarized Light: Fundamentals and Applications* (Marcel Dekker Inc., New York, 1993), p.219.
2. Dennis Derickson, *Fiber Optic Test and Measurement*, (Prentice Hall, New Jersey, 1998) p.487.
3. B.L. Heffner, "Automated measurement of polarization mode dispersion using Jones matrix eigenanalysis," *IEEE Photonics Technology Letters* **4**, 1066-1069 (1992).
4. C.D. Poole and D.L. Favin, "Polarization-mode dispersion measurements based on transmission spectra through a polarizer," *Journal of Lightwave Technology*, **12**, 917-929 (1994).

- **Calibration issues**

1. P.A. Williams, "Mode-coupled artifact standard for polarization-mode dispersion: design, assembly and implementation," *Applied Optics*, **38**, 6498-6507 (1999).
2. David J. Ives, "Calibration of a Polarisation State Analyser for Polarisation Mode Dispersion Measurements", Conference Digest, Optical Fiber Measurements Conference, Teddington, 213-216, (1997).
3. P.A. Williams, "Rotating wave-plate Stokes polarimeter for differential group delay measurements of polarization-mode dispersion," *Applied Optics*, **38**, 6508-6515 (1999).

- **PMD Uncertainty**

1. W.B. Gardner, and TIA Ad Hoc Group, "Inter-laboratory polarization mode dispersion measurement study," Technical Digest, 1994 Symposium on Optical Fiber Measurements, National Institute of Standards and Technology Special Publication 864, 171-174, (1994).
2. P.A. Williams "TIA round robin for the measurement of PMD," Technical Digest, 1996 Symposium on Optical Fiber Measurements, National Institute of Standards and Technology Special Publication 905, 155-158 (1996).
3. N. Gisin, B. Gisin, J.P. Von der Weid, R. Passy, "How Accurately Can One Measure a Statistical Quantity Like Polarization-Mode Dispersion?" *IEEE Photonics Technology Letters*, **8**, 1671-1673 (1996).
4. P.A. Williams "Accuracy issues in comparisons of time- and frequency-domain polarization mode dispersion measurements," Technical Digest, 1996 Symposium on Optical Fiber Measurements, National Institute of Standards and Technology Special Publication 905, 125-129 (1996).

Fiber and component metrology for high-speed communications

Part 2: Receiver measurements

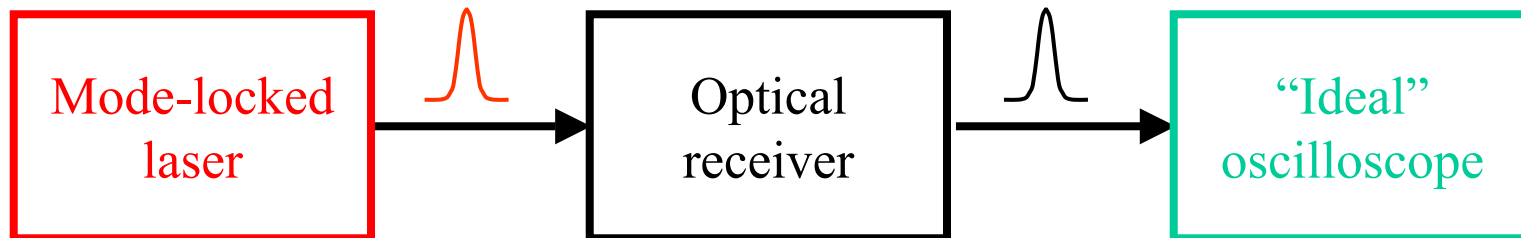
Paul Williams, [Paul D. Hale](#), and [Tracy S. Clement](#)

National Institute of Standards and Technology

Boulder, Colorado

- Use optical source with known modulation to calibrate receiver
 - Time-domain: Optical impulse
 - Frequency-domain: Sinusoidal modulation
 - Optical Noise
- Then use calibrated receiver to measure unknown modulated sources or unknown receivers

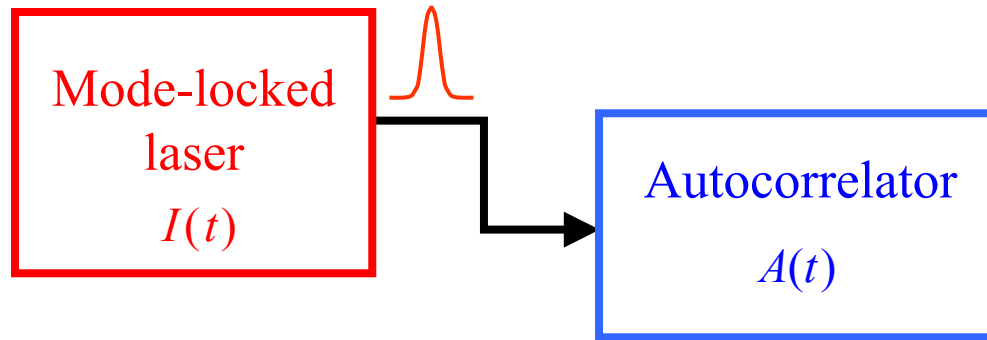
Measuring optical receiver response



$$I(t) * R(t) = M(t)$$
$$i(\omega) \cdot r(\omega) = m(\omega)$$

Receiver response is what
we want

Autocorrelation determines optical pulse shape



$$A(t) = \int_{-\infty}^{\infty} I(t + \tau) I(\tau) d\tau$$

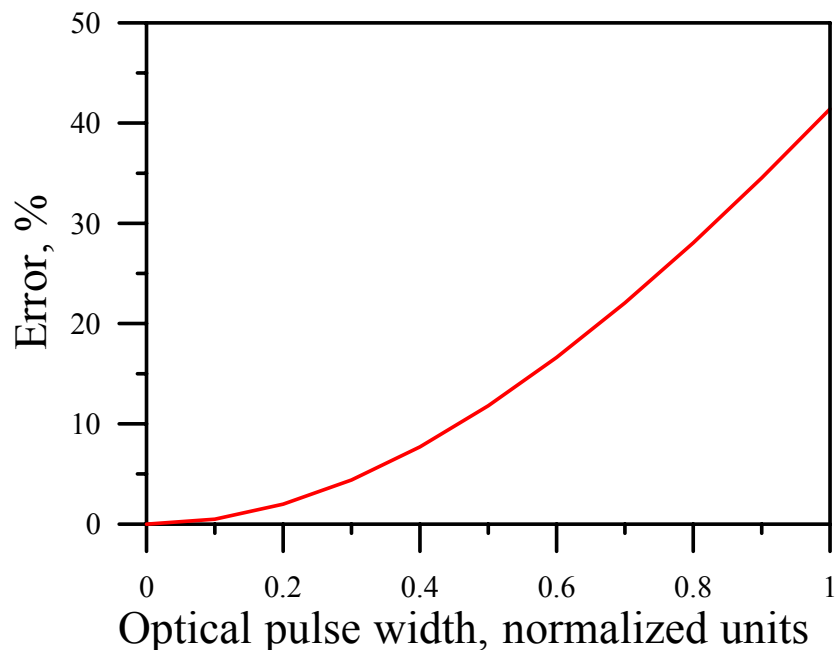
$$a(\omega) = |i(\omega)|^2$$

$$\tilde{i}(\omega) = |i(\omega)| = \sqrt{a(\omega)}$$

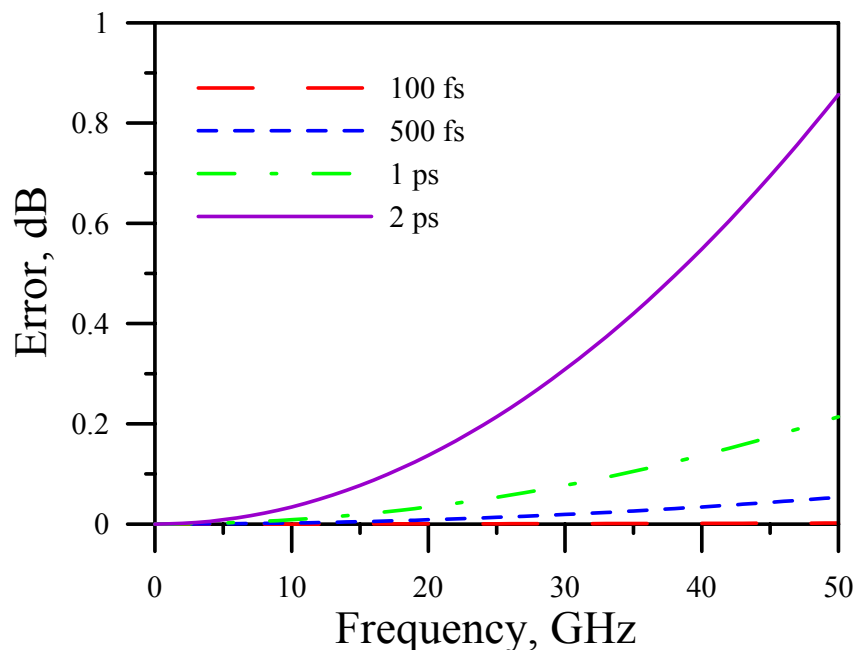
Phase error is estimated by J. Verspecht, "Quantifying the maximum phase-distortion error introduced by signal samplers," *IEEE Trans. Instrum Meas.*, **46**, 660 (1997).

Errors due to laser pulse width

Time-domain Error in FWHM



Frequency-domain Error in magnitude (dB)



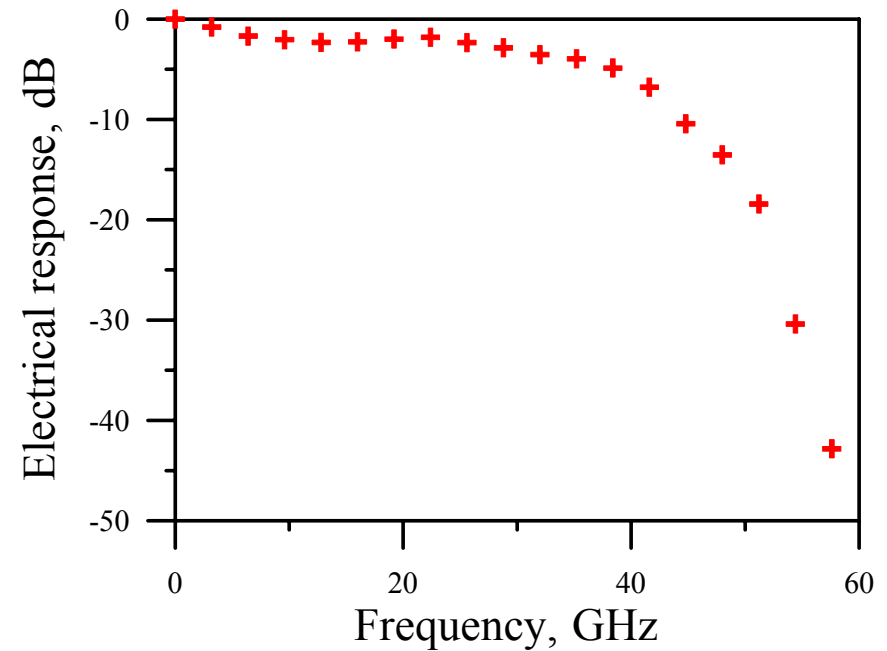
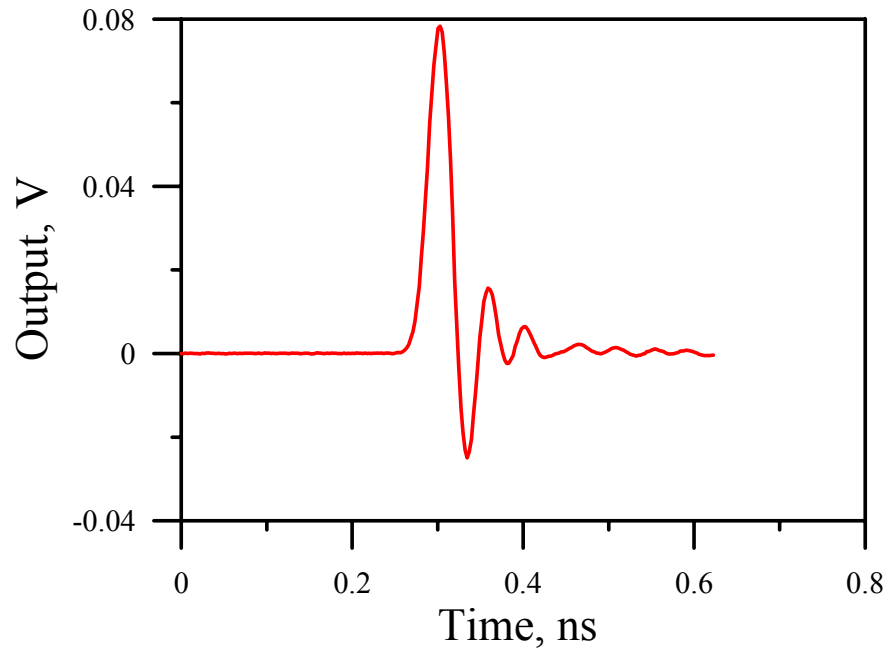
Assumes Gaussian laser pulse and Gaussian receiver response

FFT tutorial:

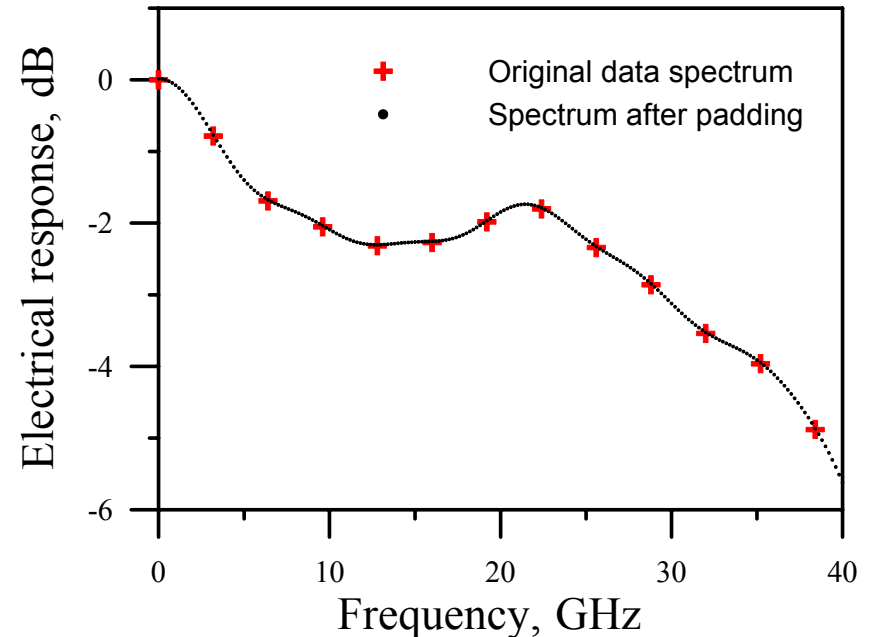
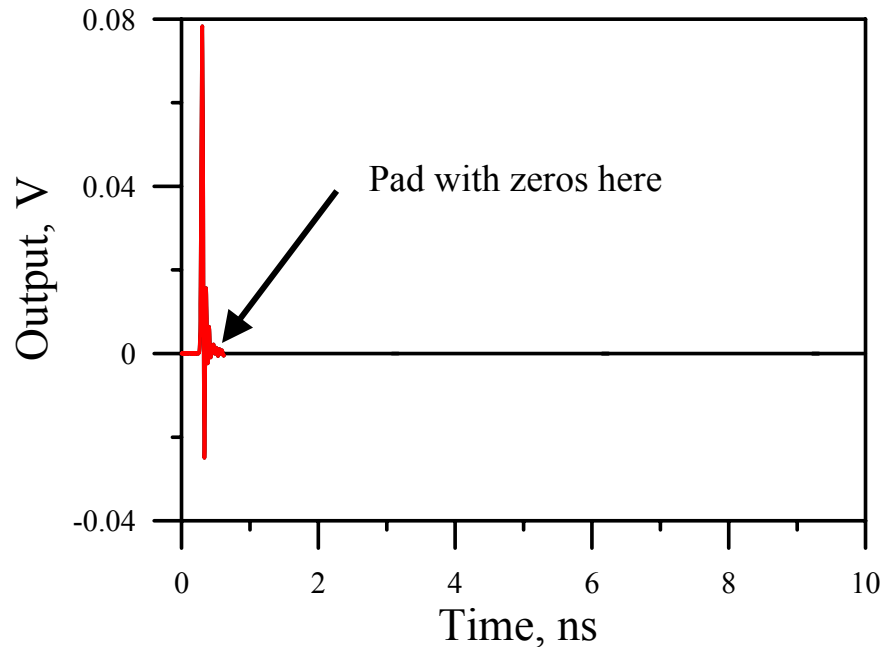
Total measured time \Rightarrow Resolution in frequency-domain

Sample spacing in time-domain \Rightarrow Highest calculated frequency

$$f_{\max} = (2\Delta t)^{-1}$$

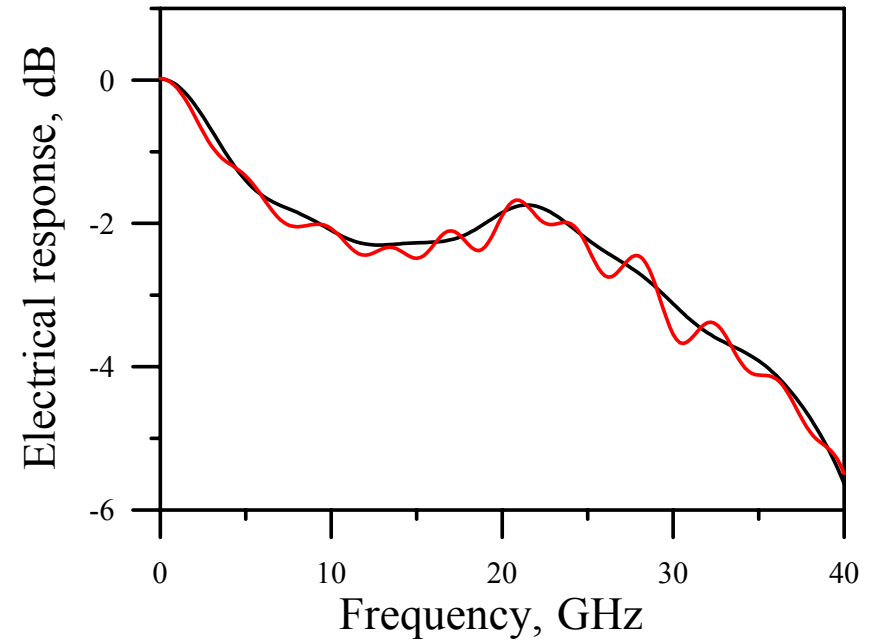
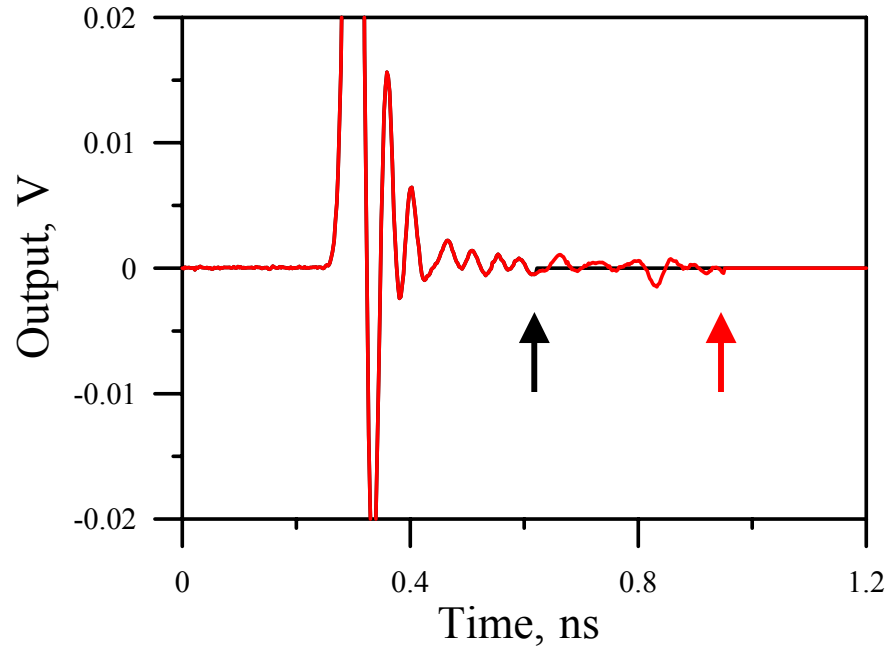


Windowed pulse + zero padding



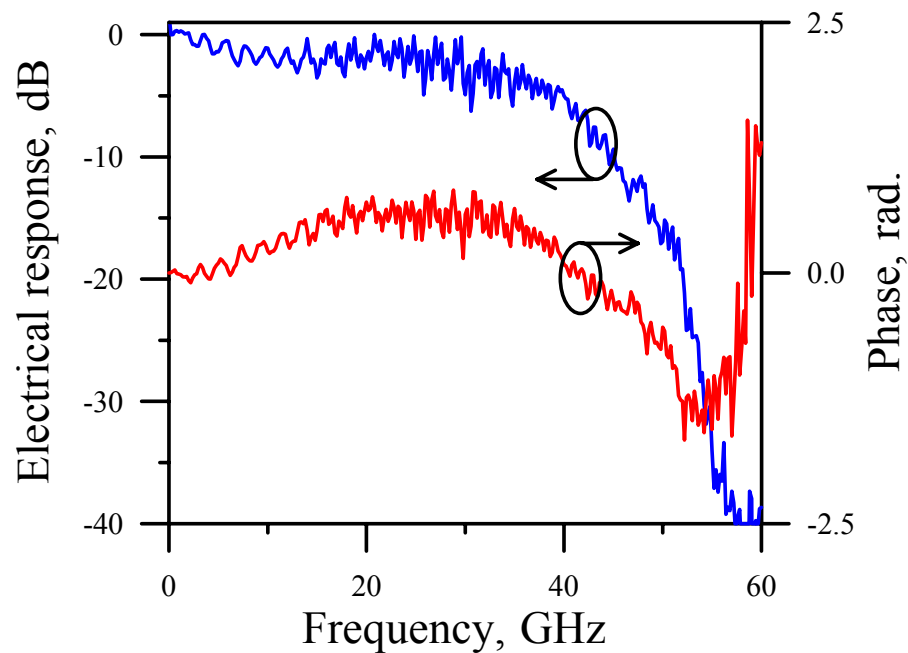
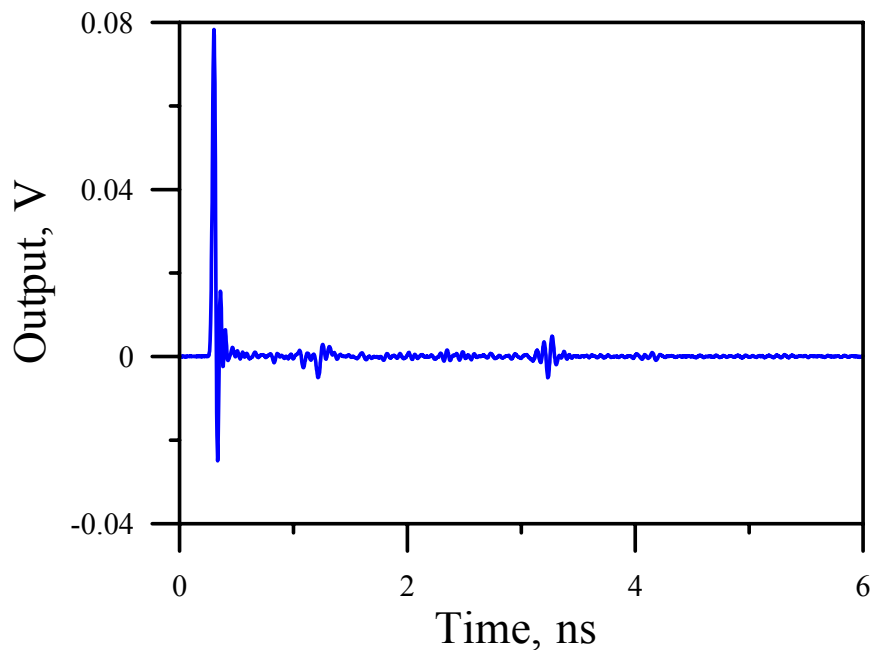
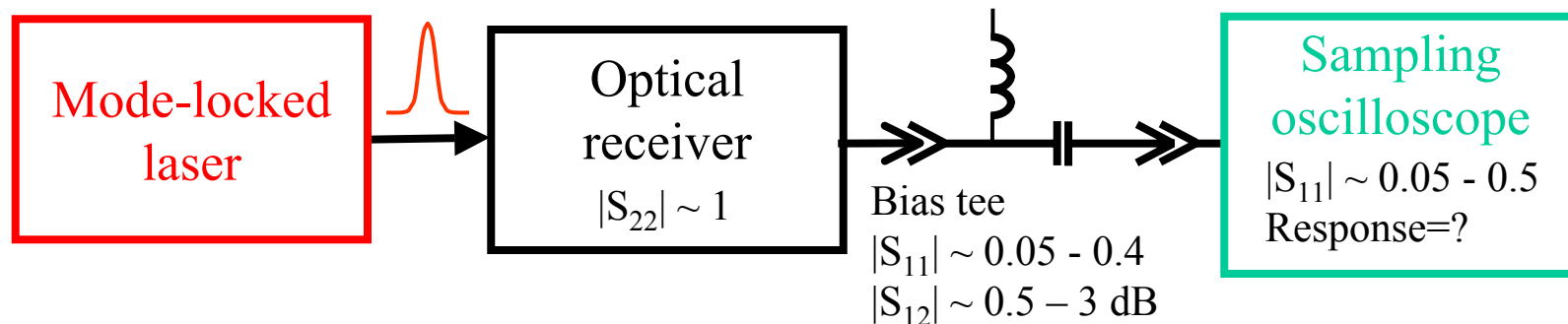
Longer time record gives frequency points spaced by 100 MHz but “real” resolution is still 3 GHz

Windowing is arbitrary!

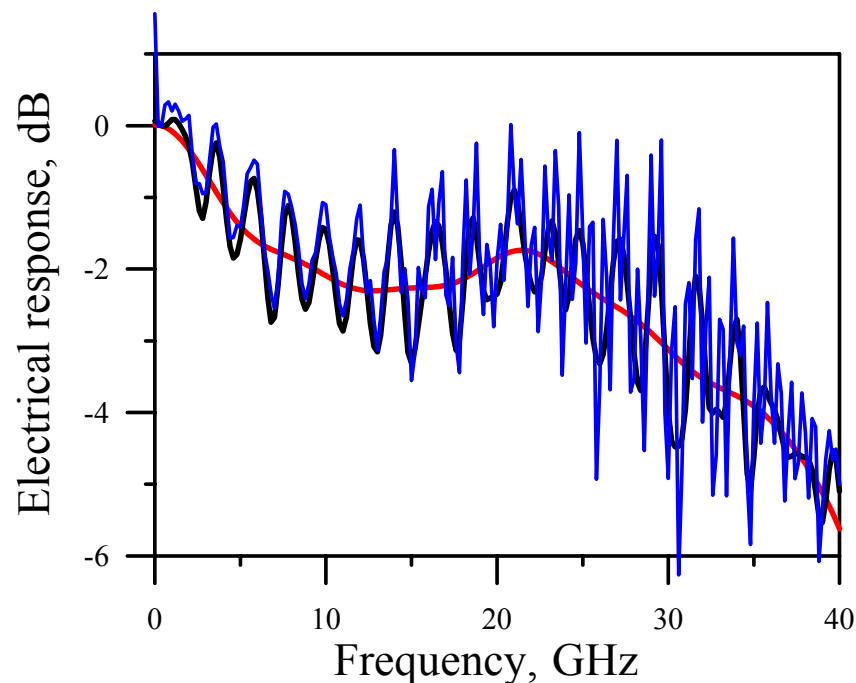
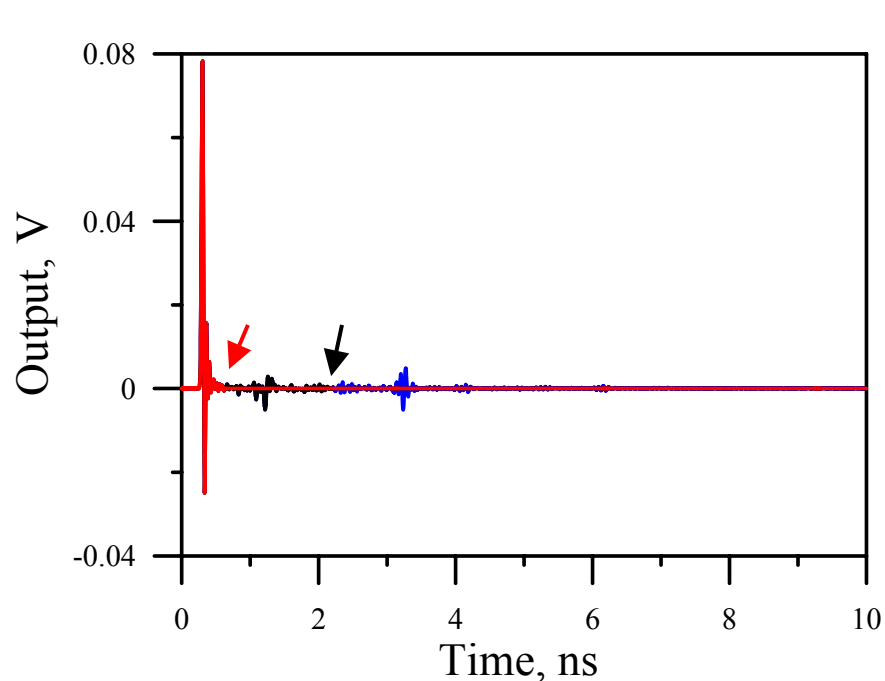


Difference in response is about 0.2 to 0.5 dB and increases with frequency

Electrical reflections and oscilloscope response



Windowing at different places gives different results

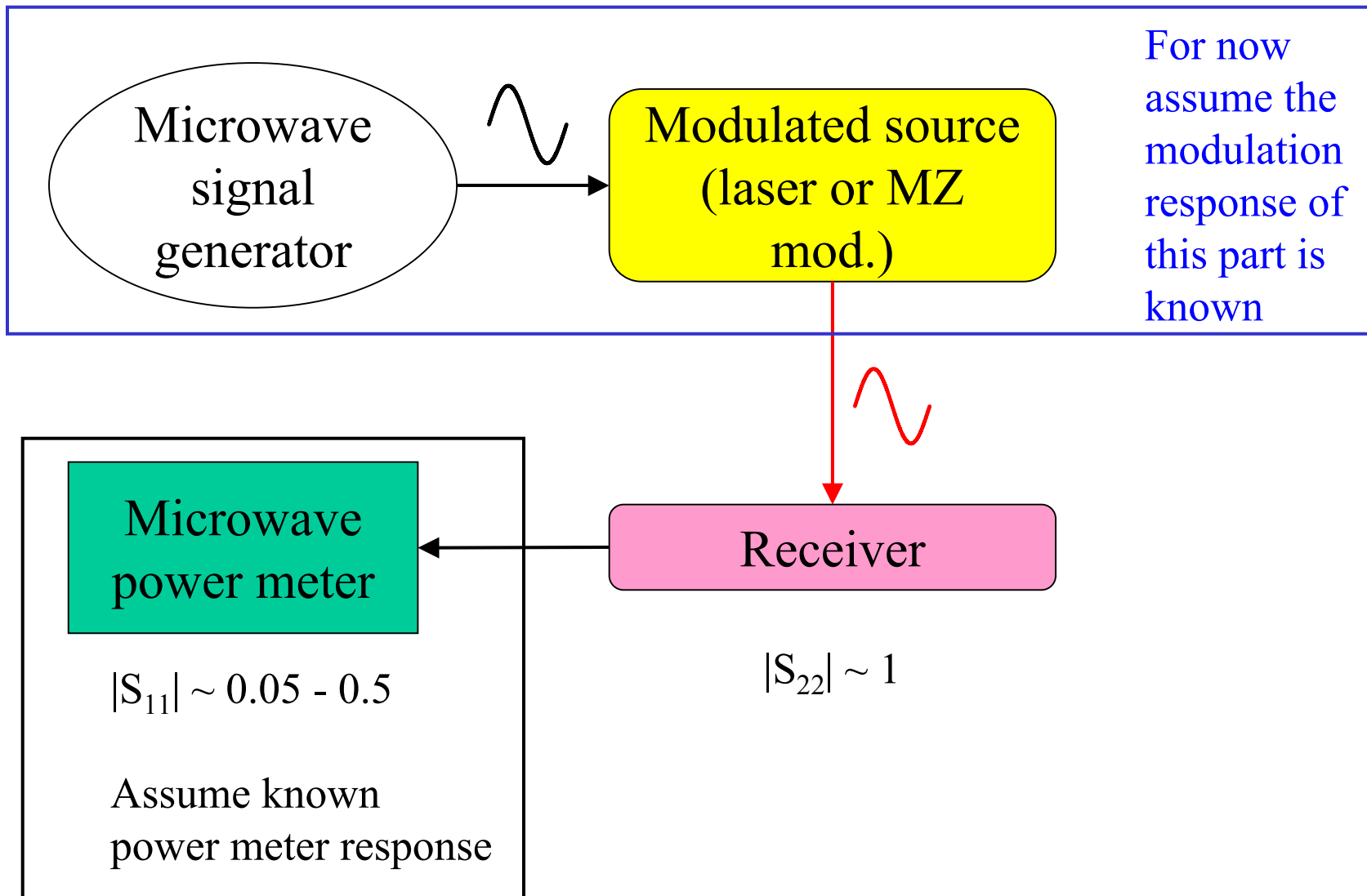


Can window out reflections from different components, but....

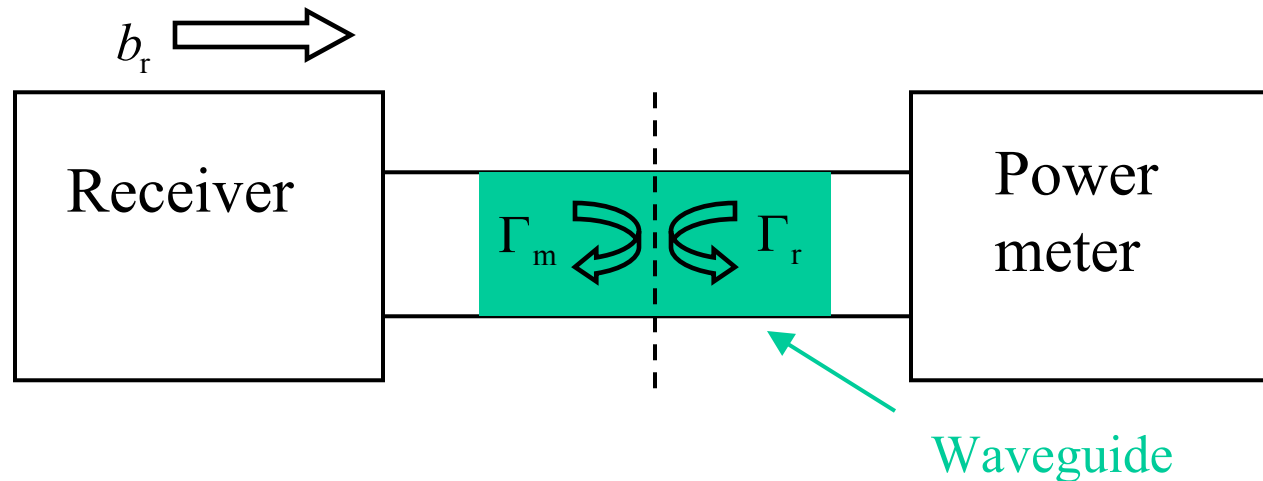
- Need detailed information on delays and component losses to choose window
- What impedance is measurement referenced to?
- ☞ Use frequency-domain microwave measurements
- Oscilloscope still needs calibration
 - Swept-sine-wave method with mismatch corrections
 - Nose-to-nose calibration
 - Transfer standard calibrated by electro-optic sampling

P. D. Hale, T. S. Clement, D. F. Williams, E. Balta, and N. D. Taneja, "Measuring the frequency response of gigabit chip photodiodes," *J. Lightwave Technol.*, **19**, 1333 (2001).

“Simple” frequency domain method



What are mismatch corrections?



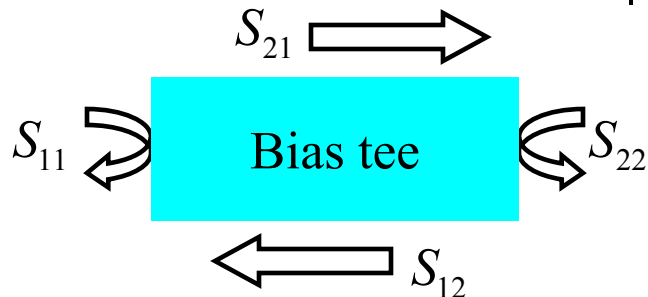
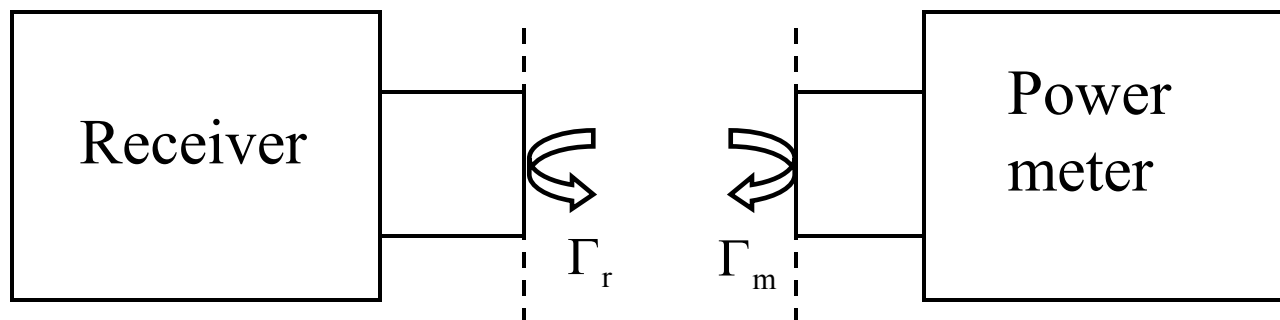
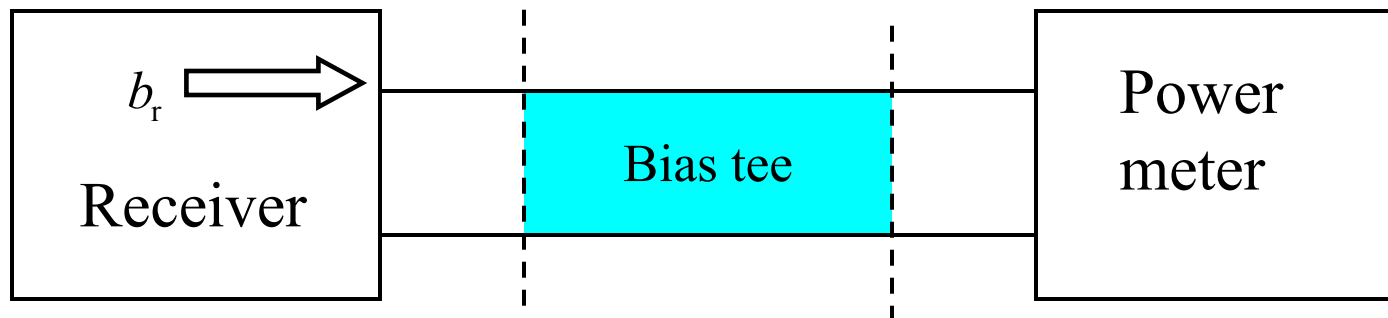
Power meter is calibrated to read incident power P_m .

$P_r = \frac{1}{2} |b_r|^2$ is power receiver would deliver to a perfect 50Ω load.

How is measured power related to properties of the receiver?

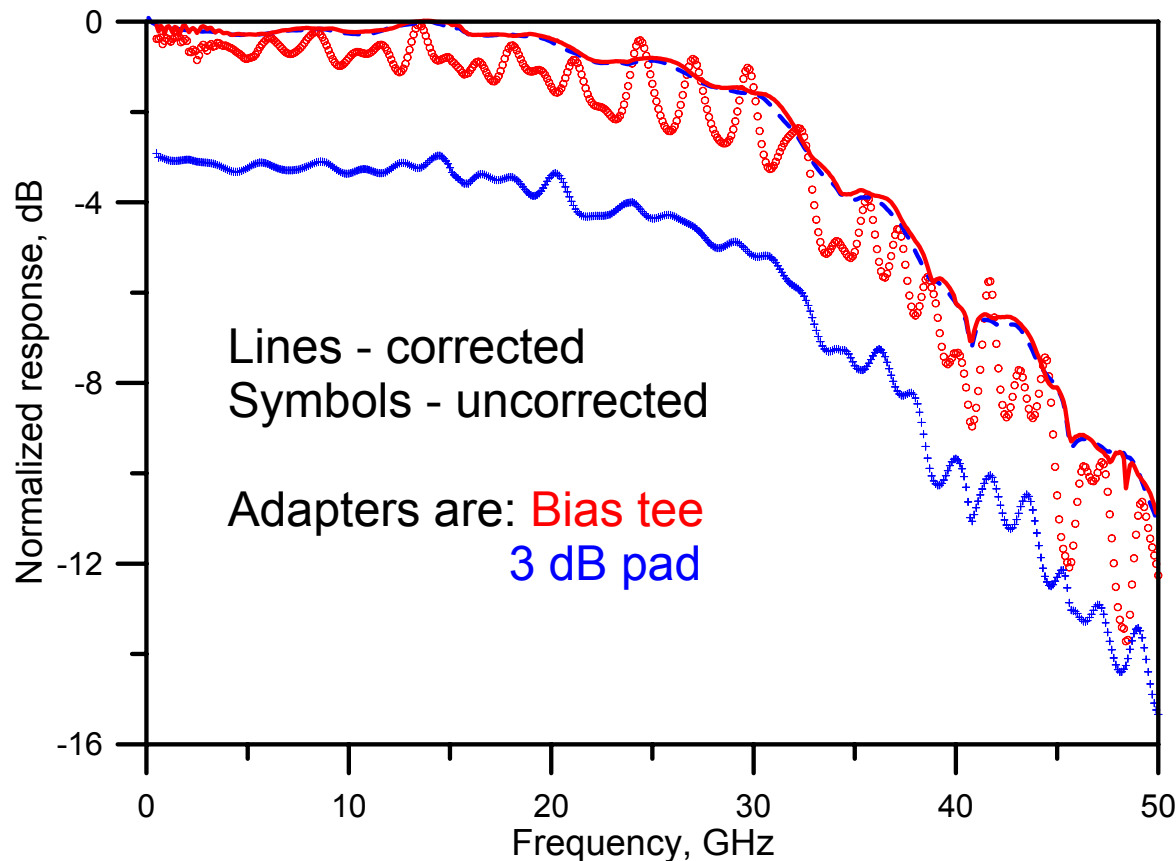
$$P_r = P_m |1 - \Gamma_r \Gamma_m|^2 = \frac{1}{2} |b_r|^2$$

What if you add a bias tee or adapter?

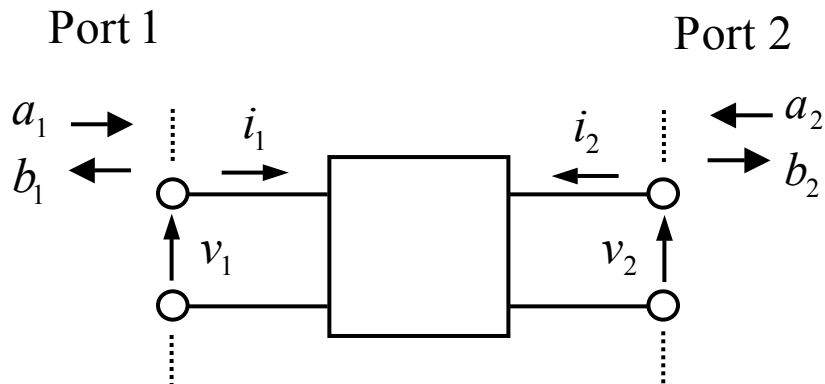


$$P_r = P_m \left| \frac{1 - S_{22}\Gamma_m - S_{11}\Gamma_r - \Gamma_r\Gamma_m (S_{12}S_{21} - S_{11}S_{22})}{S_{21}} \right|^2$$

“Mismatch” term (derived in slide #57)



- Corrections can be used for frequency-domain measurements or time-domain measurements after transformation into frequency domain.
- S-parameters are only available over a fixed frequency range, determined by the vector network analyzer.



$$a = \frac{1}{2\sqrt{Z_r}}(v + iZ_r)$$

$$b = \frac{1}{2\sqrt{Z_r}}(v - iZ_r)$$

$$P = \frac{1}{2}\text{Re}(vi^*) = \frac{1}{2}(|a|^2 - |b|^2)$$

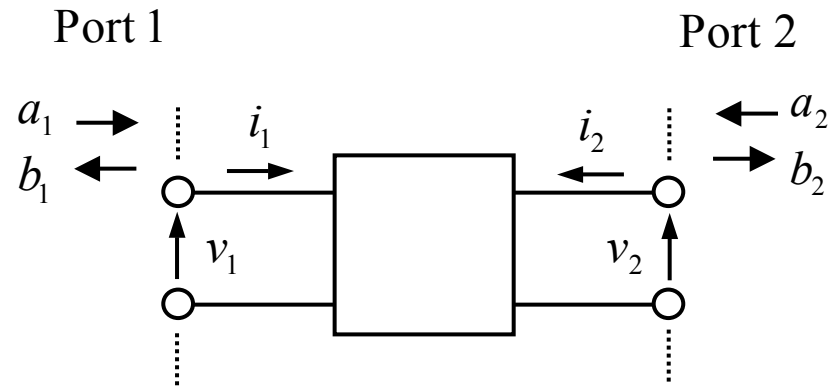
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}.$$

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

$$[T] = \frac{1}{S_{21}} \begin{bmatrix} \Delta & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

$$\Delta = S_{12}S_{21} - S_{11}S_{22}$$

Network analyzer measure S-parameters

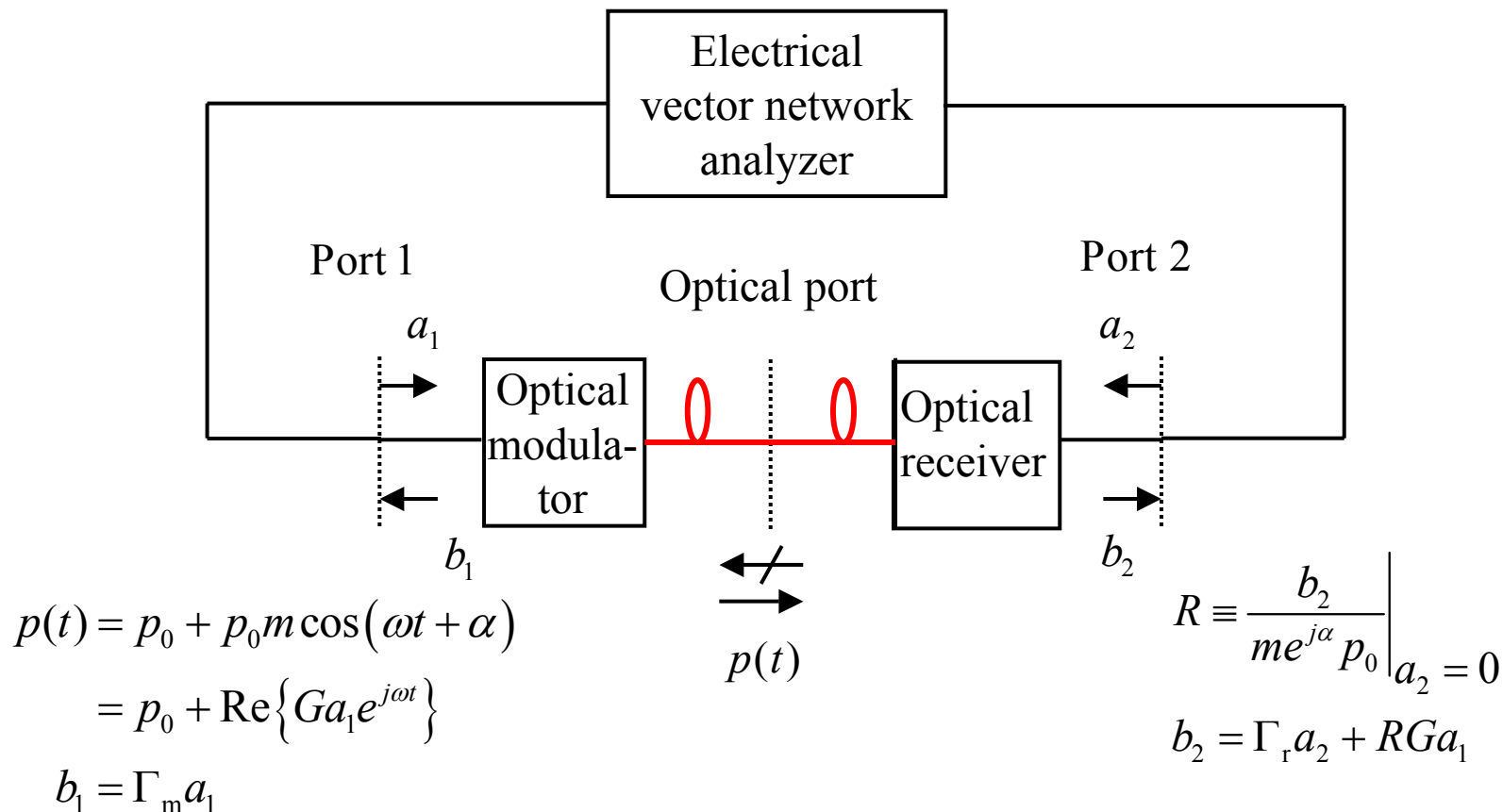


When port 2 is matched to $50\ \Omega$

$$S_{11} = b_1 / a_1$$

$$S_{21} = b_2 / a_1$$

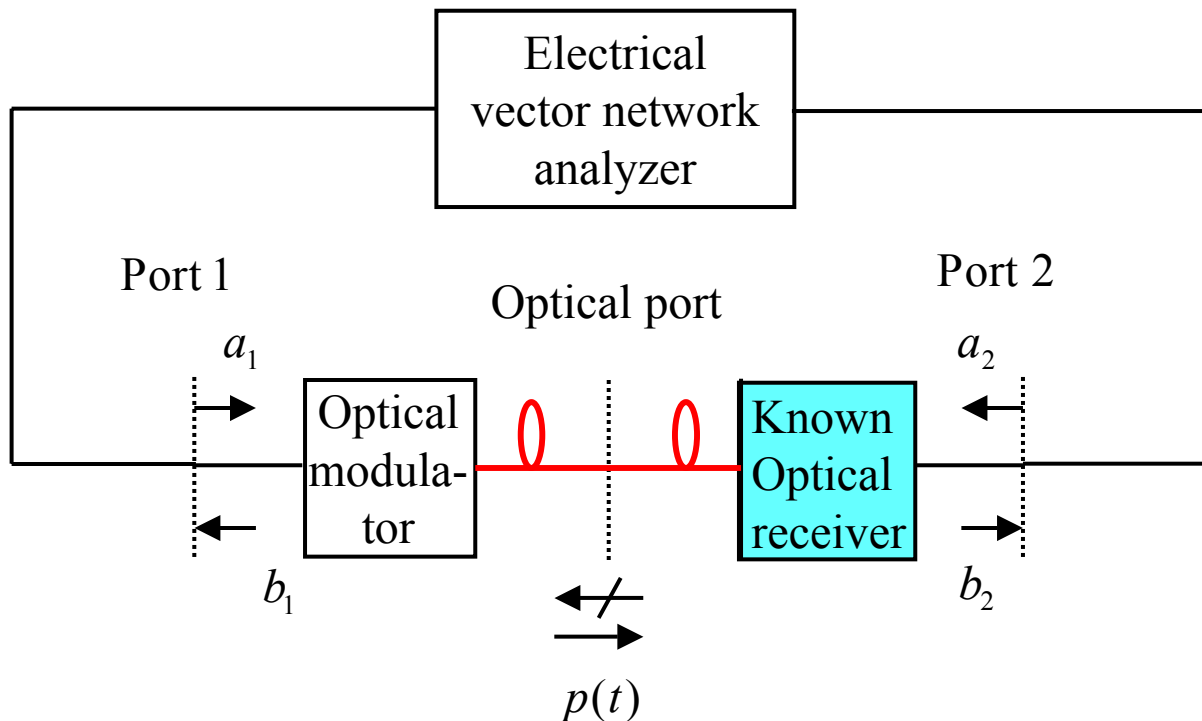
Using a VNA for optoelectronic measurements



After electrical calibration:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \Gamma_m & 0 \\ \textcolor{red}{RG} & \Gamma_r \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \equiv \begin{bmatrix} S_{11} & S_{12} \\ \textcolor{red}{S}_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Using a VNA for optoelectronic measurements



Measure unknown
modulator with
known receiver:

$$G = \frac{S_{21}}{R}$$

$$\Gamma_m = S_{11}$$

Receiver

Thevenin voltage response (use for amplified receivers)

$$\frac{v_T}{p_0 m e^{j\alpha}} \equiv \frac{v_2|_{\text{open}}}{p_0 m e^{j\alpha}} = \frac{2\sqrt{Z_r} b_2}{p_0 m e^{j\alpha}} = \frac{2\sqrt{Z_r} R}{(1 - \Gamma_r)}$$

Norton current response (use for photodiodes)

$$\frac{i_N}{p_0 m e^{j\alpha}} \equiv \frac{-i_2|_{\text{short}}}{p_0 m e^{j\alpha}} = \frac{2R}{\sqrt{Z_r} (1 + \Gamma_r)}$$

Modulator efficiency

Modulation with current input

$$\eta_i \equiv \frac{m p_0 e^{j\alpha}}{i_1} = \frac{G \sqrt{Z_r}}{1 - \Gamma_m}$$

Modulation with voltage input

$$\eta_v \equiv \frac{m p_0 e^{j\alpha}}{v_1} = \frac{G}{\sqrt{Z_r} (1 + \Gamma_m)}$$

What makes a good standard receiver?

- Operation (and characterization) at λ of interest
- No etalon effects on optical side; high optical return loss
- Precision connectors (both optical and electrical)
- Linear with high peak photocurrent
 - Example: Use mode-locked fiber laser source with 10 MHz rep. rate and pulse width of 100 fs, 1 μ A average current
 - \Rightarrow 1 A peak photocurrent for detector with infinite bandwidth
 - \Rightarrow 11 mA peak photocurrent for 50 GHz bandwidth detector
 - \Rightarrow 0.55 V peak voltage \rightarrow enough to saturate oscilloscope
 - High optical power used in modulator based system may saturate receiver
- High responsivity (A/W)
- Photodiode more stable than amplified receiver

- Photodiode calibrations or calibrated photodiodes are available from NIST, NPL, or various instrumentation or photodiode manufacturers
- Magnitude calibrated using heterodyne method
- Phase measured using electro-optic sampling or calibrated oscilloscope

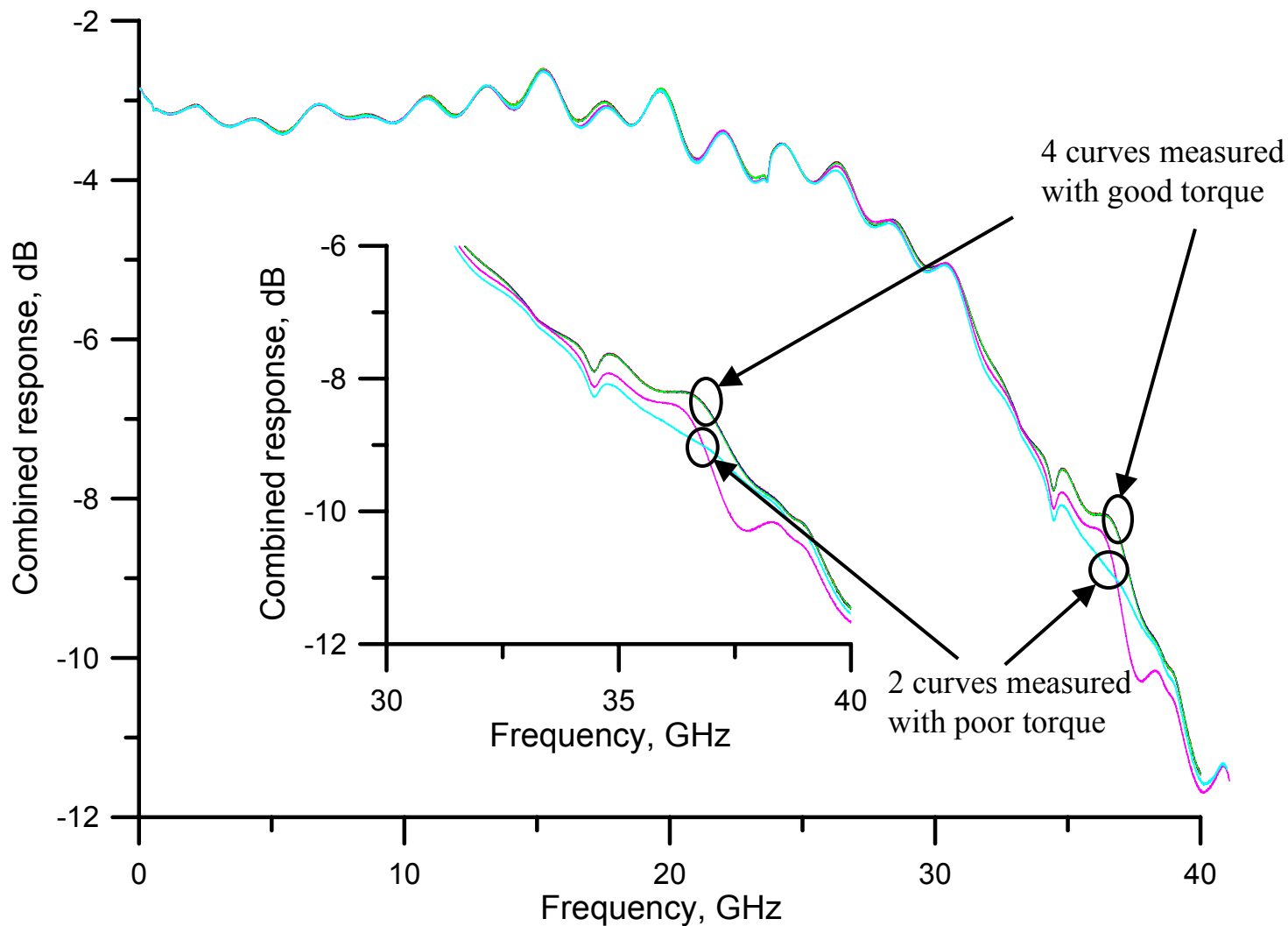
Some comments about microwave connectors

Coaxial Connectors		
Connector	Frequency range	Comments
SMA	18 GHz	Not precision connector
SMA	26.5 GHz	Sold by several companies as semi-precision connector
3.5 mm	26.5 GHz	
2.92 mm or K	40 GHz	Also OS-2.9
2.4 mm	50 GHz	Also OS-2.4 or OS-50
1.85 mm or V	65 GHz	
1mm	110 GHz	W --> W1

- Frequency range is determined by cut-off of high-order transverse modes
- Calibrated measurements are not available beyond range of single-mode operation

- Connector compatibility (at least that's what they say)
 - SMA, 3.5 mm, 2.92 mm all "compatible"
 - 2.4 mm and 1.85 mm
- *But* don't use larger male with smaller female connector - female connector may be damaged permanently!
- Blow off debris with clean "compressed air". Clean off oxide layer with solvent and cotton swab and/or toothpick
- Use connector gauges periodically
- Don't use "junk" connectors on precision components

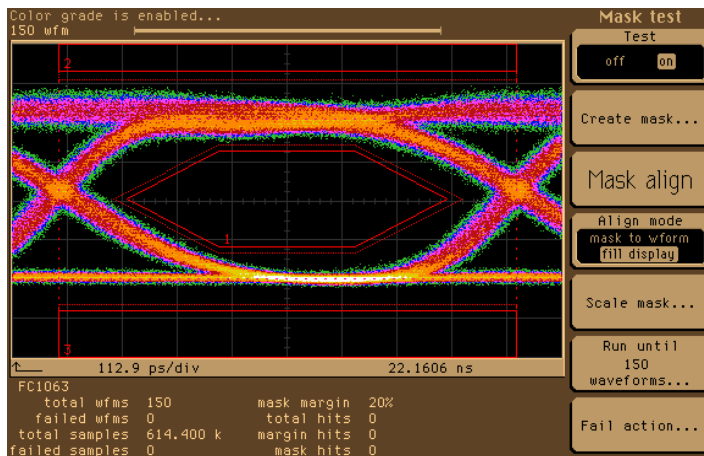
Use a torque wrench—practice makes perfect!



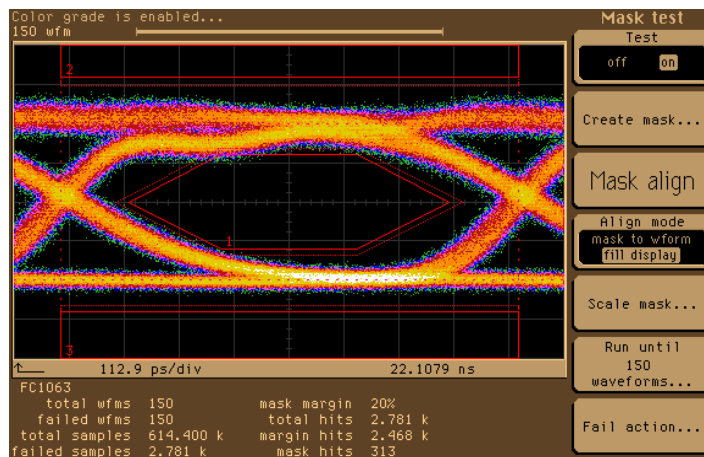
Effect of response on time-domain measurement

Measure eye-pattern for a *single* transmitter with two *different* receivers

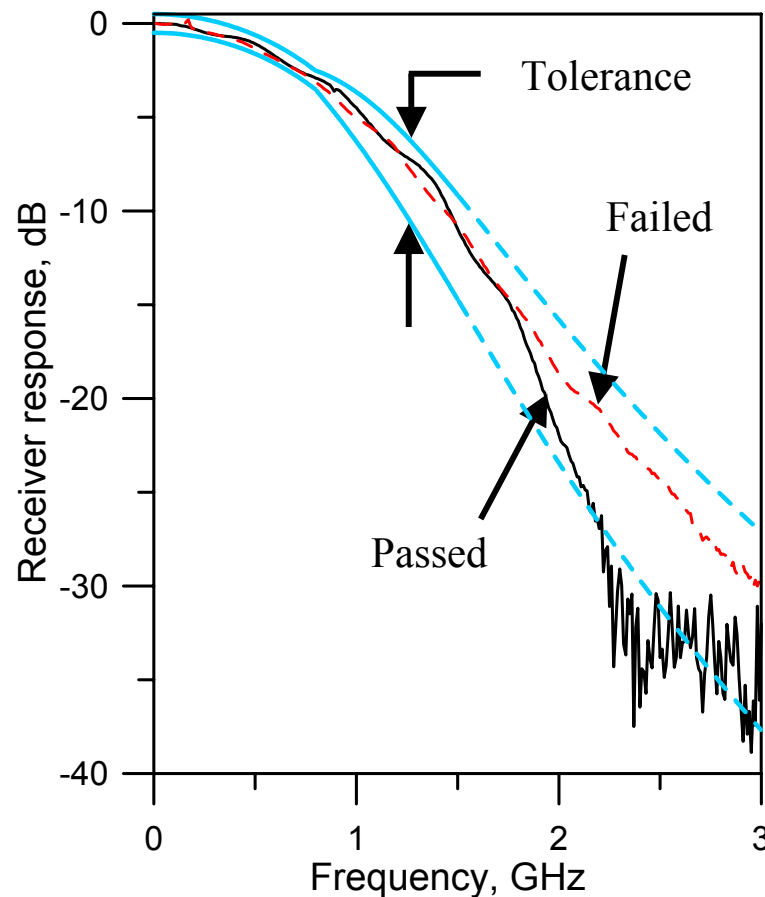
**Revr #1
PASSED**



**Revr #2
FAILED**



Receivers have same response to $>2\times$ bandwidth



- Time-domain measurements are required for digital systems
 - May be difficult to calibrate due to non-ideal properties of oscilloscope and band-limited nature of microwave measurements
- Frequency domain measurements
 - Precisely calibratable but band limited
- Need frequency domain up to 3-5× signal bandwidth for accurate time-domain representation
- Accurate and repeatable measurements require attention to good microwave practices

- **General receiver characterization**

1. P. D. Hale, T. S. Clement, D. F. Williams, E. Balta, and N. D. Taneja, "Measuring the frequency response of gigabit chip photodiodes," *J. Lightwave Technol.*, **19**, 1333 (2001).
2. P. D. Hale, T. S. Clement, D. F. Williams, "Know your response: Measuring frequency response of high-speed optical receivers," *SPIE OE Magazine*, 56, March 2001
3. P. D. Hale, T. S. Clement, and D. F. Williams, "Frequency response metrology for high-speed optoelectronic components," *OFC*, WQ1, March 2001.
4. J. E. Bowers and C. A. Burrus, "Ultrawide-band long-wavelength p-i-n photodetectors," *IEEE J. Lightwave Technol.* **5**, 1339 (1987).
5. D. J. McQuate, K. W. Chang, and C. J. Madden, "Calibration of lightwave detectors to 50 GHz," *Hewlett-Packard J.* 87 Feb. 1993.
6. J. A. Valdmanis and J. V. Rudd, "High-speed optical signals demand quick response," *Laser Focus World*, 141, March, 1995.

- **Heterodyne measurements**

1. P. D. Hale and C. M. Wang, "Heterodyne system at 850 nm for measuring photoreceiver frequency response," *Symp. Optical Fiber Meas.*, Sept. 2000.
2. P. D. Hale and C. M. Wang, *Calibration service of optoelectronic frequency response at 1319 nm for combined photodiode/rf power sensor transfer standards*, NIST SP 250-51, Dec. 1999.
3. P. D. Hale, C. M. Wang, R. Park, and W. Y. Lau, "A transfer standard for measuring photoreceiver frequency response," *J. Lightwave Technol.* **14**, 2457, (1996).
4. D. Gifford, D. A. Humphreys, and P. D. Hale, "Comparison of Photodiode frequency response measurements to 40 GHz between NPL and NIST," *Electron. Lett.* **31**, 397 (1995).

- **Oscilloscope calibration and time-domain measurements**

1. P. D. Hale, T. S. Clement, K. J. Coakley, C. M. Wang, D. C. DeGroot, and A. Verdoni, "Estimating the magnitude and phase response of a 50 GHz sampling oscilloscope using the 'nose-to-nose' method," *55th ARFTG Conference Digest*, June 2000, 35-42.
2. T. S. Clement, P. D. Hale, K. C. Coakley, and C. M. Wang, "Time-domain measurements of the frequency response of high-speed photoreceivers to 50 GHz," *Symp. Optical Fiber Meas.*, Sept. 2000.
3. R. T. Hawkins, M. D. Jones, S. H. Pepper, J. H. Goll, "Comparison of fast photodetector response measurements by optical heterodyne and pulse response techniques," *IEEE J. Lightwave Technol.*, **9**, 1289 (1991).
4. R. T. Hawkins, M. D. Jones, S. H. Pepper, J. H. Goll, and M. K. Ravel, "Vector characterization of photodiodes, photoreceivers, and optical pulse sources by time-domain pulse response measurements," *IEEE Trans. Instrum. Meas.*, **41**, 467 (1992).
5. K. J. Coakley and P. D. Hale, "Alignment of noisy signals," *IEEE Trans. Instrum. Meas.*, **50**, 141 (2001).
6. C. M. Wang, P. D. Hale, and K. J. Coakley, "Least-squares estimation of time-base distortion of sampling oscilloscopes," *IEEE Trans. Instrum. Meas.*, **48**, 1324 (1999).
7. C. M. Wang, P. D. Hale, K. J. Coakley, and T. S. Clement, "Uncertainty of oscilloscope time-base distortion estimates," *IEEE Trans. Instrum. Meas.*, Feb., (2002).
8. K. A. Remley, D. F. Williams, D. C. DeGroot, J. Verspecht, J. Kerley, "Effects of nonlinear diode junction capacitance on the nose-to-nose calibration," *IEEE Microwave Wireless Comp.Lett.*, **11**, 196 (2001).
9. J. Verspecht, "Quantifying the maximum phase-distortion error introduced by signal samplers," *IEEE Trans. Instrum Meas.*, **46**, 660 (1997).

- **Electro-optic sampling**

1. D. F. Williams, P. D. Hale, T. S. Clement, and J. M. Morgan, “Calibrating electro-optic sampling systems,” *IMS Conference Digest*, 1473, May 2001.
2. D. F. Williams, P. D. Hale, T. S. Clement, and J. M. Morgan, “Mismatch corrections for electro-optic sampling systems” *56th ARFTG Conference Digest*, 141, Dec. 2000.

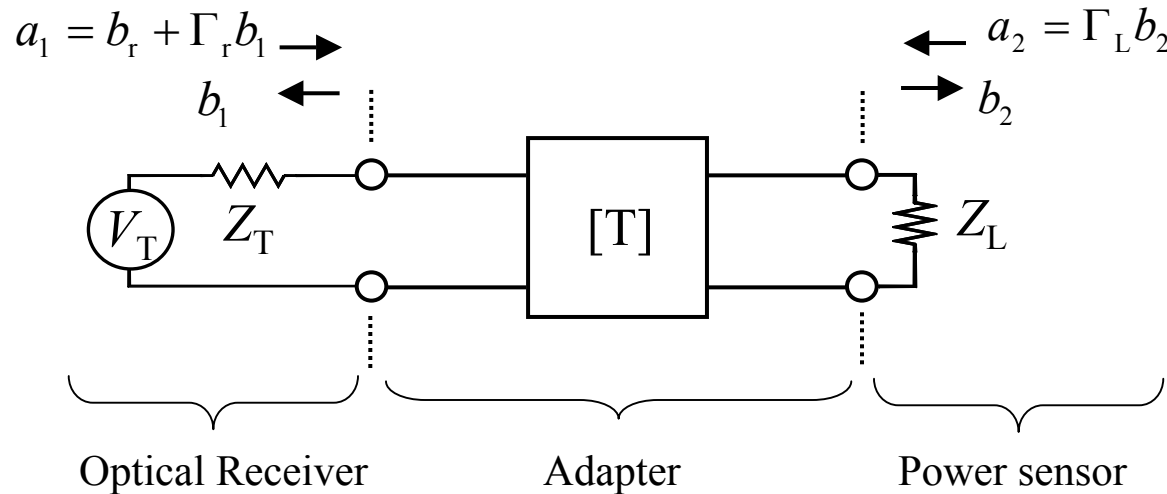
- **Modulator and modulator-based measurements**

1. D. A. Humphreys, “Integrated-optic system for high-speed photodetector bandwidth measurements,” *Electron. Lett.*, **25**, 1555 (1989).
2. S. Uehara, “Calibration of optical modulator frequency response with application to signal level control,” *Appl. Opt.*, **17**, 68 (1978).
3. B. H. Kolner and D. W. Dolfi, “Intermodulation distortion and compression in an integrated electrooptic modulator,” *Appl. Opt.*, **26**, 3676 (1987).

- **Receiver measurements using optical noise**

1. D. M. Baney, W. V. Sorin, and S. A. Newton, “High-frequency photodiode characterization using a filtered intensity noise technique,” *IEEE Photon. Technol. Lett.*, **6**, 1258 (1994).
2. G. E. Obarski and J. D. Splett, “Transfer standard for the spectral density of relative intensity noise of optical fiber sources near 1550 nm,” *J. Opt. Soc. Am. B*, **18**, 750 (2001).

De-embedding using T matrix



$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = [T] \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

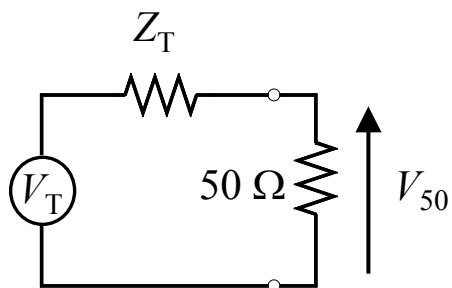
$$\begin{bmatrix} b_1 \\ b_r + \Gamma_r b_1 \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} \Delta & S_{11} \\ -S_{22} & 1 \end{bmatrix} \begin{bmatrix} b_2 \Gamma_L \\ b_2 \end{bmatrix};$$

solving gives

$$\frac{b_r}{b_2} = \frac{1}{S_{21}} (1 - \Gamma_L S_{22} - \Gamma_r S_{11} - \Gamma_L \Gamma_r \Delta) \Rightarrow P_r = \frac{1}{|S_{21}|^2} |1 - \Gamma_L S_{22} - \Gamma_r S_{11} - \Gamma_L \Gamma_r \Delta|^2 P_m$$

Relating measurements to equivalent circuit models

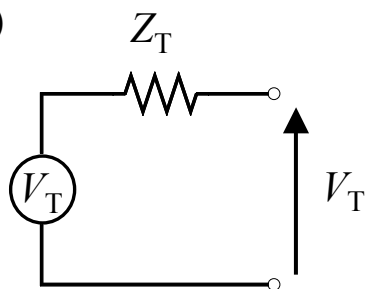
(a)



Voltage delivered to $50\ \Omega$

$$V_{50} = \sqrt{Z_0} b_r = \sqrt{2Z_0 P_r} \text{ (neglecting } \arg(b_r))$$

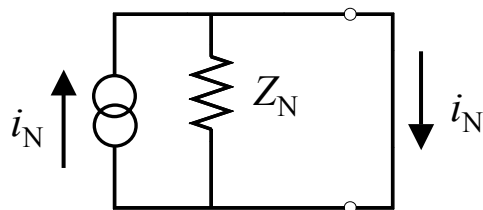
(b)



Thevenin equivalent voltage

$$V_T = \frac{Z_T + Z_0}{Z_0} V_{50}$$

(c)

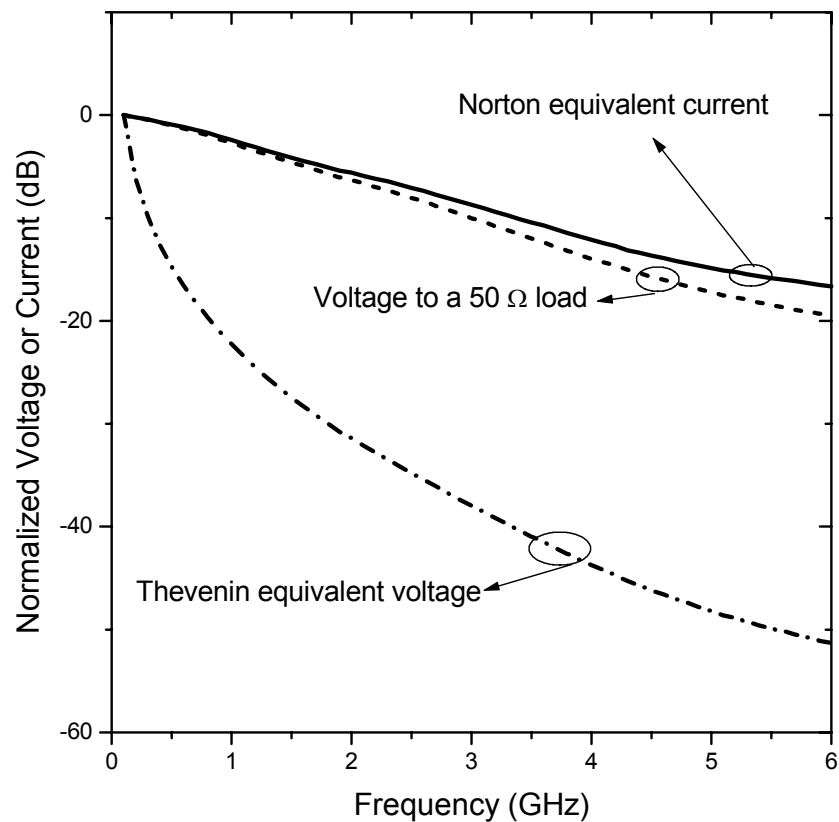
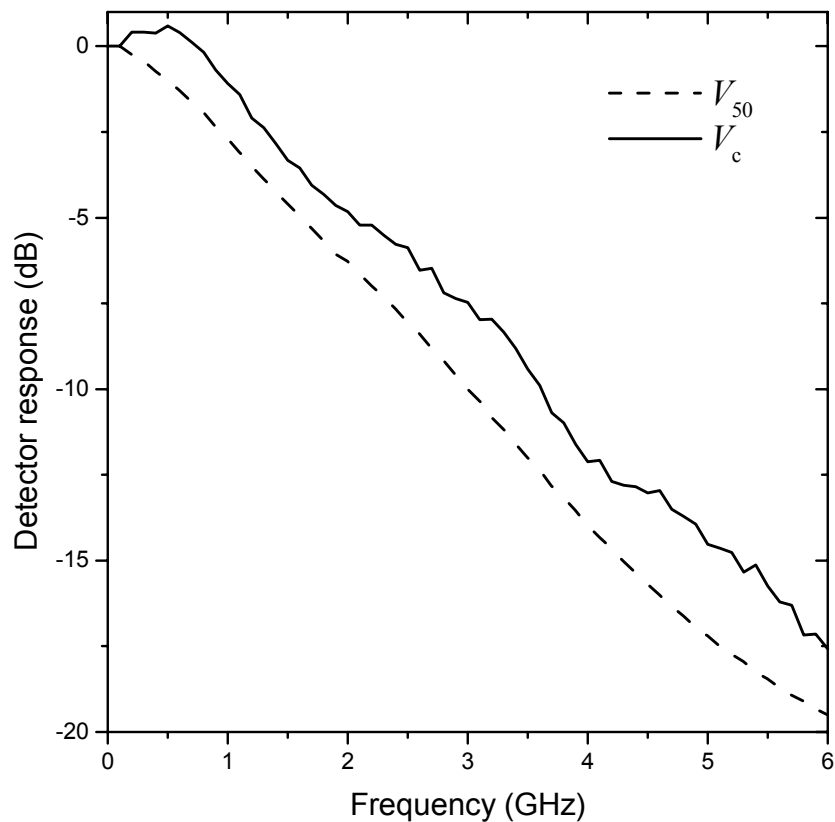


Norton equivalent current

$$i_N = \frac{V_T}{Z_N}$$

$Z_0 = \text{reference impedance} = 50\ \Omega$

Response of different equivalent sources



P. D. Hale, T. S. Clement, D. F. Williams, E. Balta, and N. D. Taneja, "Measuring the frequency response of gigabit ship photodiodes," *J. Lightwave Technol.*, **19**, 1333 (2001).